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MATHEMATICS

Polynomials

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Do you know how much percentage of marks a student has to score in IITJEE advanced exam to get a rank in top 100 ? a seat getting rank in IITs ?

Unbelievably they are just 75% and 35% (avg of last 10 years data)
Many students who score 10/10 in boards are not able to score 35% in
JEE advanced but an average student can make it to IIT if the coaching institute has

- ✓ Expert faculty - Excellent teaching
- ✓ Meticulous planning - Long-term strategy
- ✓ Constant motivation - Continuous support
- ✓ Rigorous practice - Accuracy and Speed

Previous Years JEE ADVANCED Rank VS % of Marks

RANK	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	Avg %	2021 (M. Marks 360) Marks	2021 (M. Marks 360) Rank	2021 (M. Marks 360) Marks	2021 (M. Marks 360) Rank
1	85.48	91.67	96.01	92.22	92.78	93.06	86.02	92.62	93.61	93.01	88.89	91.40	348	1	196	1544
10	81.39	85.42	88.48	85.83	86.94	83.92	78.49	86.88	86.11	82.80	81.06	84.99	324	8	185	2043
100	70.76	79.79	78.68	76.39	77.50	69.84	68.61	83.33	74.72	72.85	71.72	74.93	323	13	178	2395
500	62.17	72.08	69.36	66.67	68.61	60.32	54.57	77.32	64.44	61.02	59.34	65.08	313	25	160	3542
1000	57.87	68.13	64.46	61.67	63.33	56.75	49.46	72.95	58.59	55.65	53.28	60.22	310	27	154	4137
2000	52.97	62.50	58.33	55.56	57.22	49.21	43.55	67.48	53.06	50.00	46.46	54.21	302	46	145	5005
3000	49.69	59.17	54.41	51.94	53.61	45.44	42.74	63.38	49.17	46.77	42.42	50.79	297	66	135	6180
4000	47.03	56.67	51.72	49.17	51.11	42.66	37.90	60.38	46.39	44.09	39.39	47.86	296	70	131	6750
5000	44.99	54.38	49.51	47.22	48.89	40.48	36.02	57.65	43.89	41.94	37.12	45.64	286	94	126	7516
6000	43.35	52.17	47.55	45.56	47.22	38.69	34.41	55.45	41.94	40.32	35.10	43.85	266	201	121	8494
7000	41.92	51.04	45.83	43.89	45.56	37.10	33.33	53.55	40.28	38.71	33.59	42.25	254	299	109	10848
8000	40.49	49.79	43.38	42.50	44.44	35.91	31.99	51.63	38.61	37.63	32.32	40.79	245	395	102	12874
9000	39.47	48.54	43.14	41.11	43.06	34.52	30.91	50.27	37.22	36.29	30.81	39.58	239	471	100	13215
10000	38.85	47.71	41.91	40.00	41.94	33.33	29.84	48.90	36.11	35.22	29.80	38.51	231	600	85	18547
QUAL%	38.85	47.71	34.55	33.88	35.00	23.81	20.16	22.50	25.00	25.00	17.42	29.44	212	1006	79	21157

2 Polynomials

KEY FACTS

1. A function $f(x)$ of the form $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$
where $a_0, a_1, a_2, ..., a_n$ are real numbers, $a_n \neq 0$ and n is a non negative integer is called a polynomial in x.
The real numbers $a_0, a_1, a_2, ..., a_n$ are called coefficients of the polynomial.

Ex. (*a*) $6x^2 - 8x + 5$ is a polynomial with integral coefficients.

(*b*) $\frac{9}{5}x^3 + \frac{4}{7}x^2 - 8$ is a polynomial with rational coefficients.

(*c*) $6x^4 - \sqrt{3}x^2 + 3\sqrt{5}$ is a polynomial with real coefficients.

2. Types of Polynomials

- **Monomial :** A polynomial having only one term as $9, \sqrt{2}x, \frac{1}{4}x^2$, etc.
- **Binomial :** A polynomial having only two terms as $4x - 5$, $6x^2 + 8x$, etc.
- **Trinomial :** A polynomial having only three terms as $4x^2 - \sqrt{2}x + \frac{1}{3}$

3. Degree of a Polynomial :

- The degree of a polynomial in one variable is the highest exponent of the variable in that polynomial.
  Degree of $9x^7 - 6x^5 + 4x^3 + 8$ is **7.**
- The degree of a polynomial in more than one variable is the highest sum of the powers of the variables.
  Degree of $4x^5 - 6x^2y^4 + 8 - 3xy^6$ is **1 + 6 = 7.**
- A polynomial is said to be ***linear***, ***quadratic***, ***cubic*** or ***biquadratic*** if its degree is 1, 2, 3 or 4 respectively.
- A **constant** is a polynomial of degree 0.

4. Division of a Polynomial by Another Polynomial.

If $f(x)$ and $g(x)$ are two polynomials, $g(x) \neq 0$, such that $f(x) = g(x).\ q(x) + r(x)$ where degree of $r(x)$ < degree of $f(x)$, then $f(x)$ is divided by $g(x)$, and it gives $\boldsymbol{q(x)}$ as **quotient** and $\boldsymbol{r(x)}$ as **remainder**.

Note : If $r(x) = 0$, then the divisor $g(x)$ is a factor of $f(x)$.

5. Remainder Theorem : If $\boldsymbol{f(x)}$ be any polynomial of degree ≥ 1, and a be any number, then if $f(x)$ is divided by $(x - a)$, the remainder is $\boldsymbol{f(a)}$.

Ex. (*a*) The remainder when $f(x) = (5x^2 - 4x - 1)$ is divided by $(x - 1)$ is $f(1) = 5.1^2 - 4.1 - 1 = \mathbf{0}$.

(*b*) The remainder when $f(x) = x^4 + 2x^3 - 3$ is divided by $(x + 2)$ is $f(-2) = (-2)^4 + 2.(-2)^3 - 3 = 16 - 16 - 3 = \mathbf{-3}$.

6. Factor Theorem : Let $f(x)$ be a polynomial of degree $n > 0$. If $f(a) = 0$, for any real number a, then $(x - a)$ is a factor of $f(x)$.
Conversely, if $(x - a)$ is a factor of $f(x)$, then $f(a) = 0$.

Ex. $f(x) = x^3 - 6x^2 + 11x - 6$ is exactly divisible by $(x - 1)$ as $f(1) = 1^3 - 6.1^2 + 11.1 - 6 = \mathbf{0}$.

7. Some useful Identities :

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$
- $(a + b)(a - b) = a^2 - b^2$
- $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
- $(a + b)^2 - (a - b)^2 = 4ab$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
- $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $(a^3 + b^3 + c^3) - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - ca - bc)$
- $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$
- $(a^n - b^n)$ is divisible by $(a - b)$ for all values of n.
- $(a^n - b^n)$ is divisible by $(a + b)$ for only even values of n.
- $(a^n + b^n)$ is never divisible by $(a - b)$
- $(a^n + b^n)$ is divisible by $(a + b)$ only when n is odd.

8. Note : When a polynomial $f(x)$ is divided by $(x - a)$ and $(x - b)$, the respective remainders are A and B. Then, if the same polynomial is divided by $(x - a)(x - b)$, then the remainder will be :

$$\frac{A - B}{a - b}x + \frac{Ba - Ab}{a - b}.$$

9. Homogeneous Expressions : If all the terms of an algebraic expression are of the same degree, then such an expression is called homogeneous expression.

- Homogeneous expressions in (x, y) of
  Degree 1 $\rightarrow px + qy$
  Degree 2 $\rightarrow px^2 + rxy + qy^2$
  Degree 3 $\rightarrow px^3 + rx^2y + sxy^2 + qy^3$
- Homogeneous expression in (x, y, z) of
  Degree 1 $\rightarrow px + qy + rz$
  Degree 2 $\rightarrow px^2 + qy^2 + rz^2 + sxy + tyz + uzx$
  Degree 3 $\rightarrow ax^3 + by^3 + cz^3 + dx^2y + exy^2 + fy^2z + gyz^2 + hz^2x + kzx^2$.

10. Symmetric Expression : An algebraic expression $f(x, y)$ in two variables x, y is called a symmetric expression if $f(x, y) = f(y, x)$.

An algebraic expression $f(x, y, z)$ is said to be a **cyclic expression,** if $f(x, y, z) = f(y, z, x) = f(z, x, y)$

e.g. $f(a, b, c) = a(b - c) + b(c - a) + c(a - b)$

- $\sum$ **(Sigma)** is used for the **sum** of the terms of a cyclic expression.

  Ex. $\sum_{x,\, y,\, z} x^3(y - z) = x^3(y - z) + y^3(z - x) + z^3(x - y)$

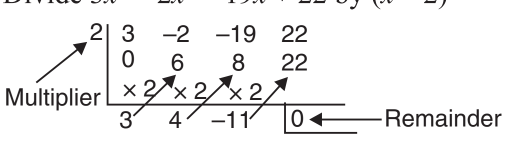
- π **(Pi)** is used for the **product** of the terms of a cyclic expression.

  Ex. $\pi_{a,b,c}\,(a - b) = (a - b)(b - c)(c - a)$

11. Horner's Method of Synthetic Division for Factorization

Ex. Divide $3x^3 - 2x^2 - 19x + 22$ by $(x - 2)$

Sol.



$\therefore f(x) = (x - 2)(3x^2 + 4x - 11)$

Step 1: Write the coefficients of the descending powers of x in the first horizontal row.

Step 2: The multiplier is obtained by putting the divisor $(x - 2) = 0 \Rightarrow x = 2$.

Step 3: Now below the 1st coefficient, *i.e.*, 3 in the first horizontal row, put 0 and add 3 + 0, *i.e.*, 3.

Now 3 × multiplier = 3 × 2 = 6 = 2nd element of 2nd horizontal row. –2 + 6 = 4

Now 4 × multiplier = 4 × 2 = 8 = 3rd element of 2nd horizontal row. –19 + 8 = – 11

For the last element again $-11 \times 2 = 22$ and $22 + (-22) = 0$.

The first three figures in the third row stand for the coefficients of descending powers of x of quotient and the last entry is for the remainder.

SOLVED EXAMPLES

Ex. 1. For what value of p is the coefficient of x^2 in the product $(2x-1)(x-k)(px+1)$ equal to 0 and the constant term equal to 2 ? ***(CDS 2005)***

Sol. $(2x-1)(x-k)(px+1) = (2x-1)(px^2 + x - kpx - k)$

$$= 2px^3 + 2x^2 - 2kpx^2 - 2kx - px^2 - x + kpx + k$$

$$= 2px^3 + x^2[2 - 2kp - p] - x[2k + 1 - kp] + k$$

Here constant term $= k = \mathbf{2}$.

Coefficient of $x^2 = 2 - 2kp - p = 2 - 4p - p = 2 - 5p$

Given, $2 - 5p = 0 \Rightarrow p = \mathbf{\frac{2}{5}}$.

Ex. 2. For what value of m will the expression $3x^3 + mx^2 + 4x - 4m$ be divisible by $x + 2$? ***(CDS 2005)***

Sol. $f(x) = 3x^3 + mx^2 + 4x - 4m$

$f(x)$ is divisible by $(x + 2)$ if $f(-2) = 0$

Now $f(-2) = 3(-2)^3 + m(-2)^2 + 4(-2) - 4m = -24 + 4m - 8 - 4m = -32 \neq 0$

$\therefore$ No such value of m exists for which $(x + 2)$ is a factor of the given expression.

Ex. 3. If $x^5 - 9x^2 + 12x - 14$ is divisible by $(x - 3)$, what is the remainder ? ***(CDS 2011)***

Sol. Let $f(x) = x^5 - 9x^2 + 12x - 14$

$f(x)$ is divisible by $(x - 3)$ so remainder $= f(3)$.

$\therefore f(3) = (3)^5 - 9(3)^2 + 12(3) - 14 = 243 - 81 + 36 - 14 = \mathbf{184}$.

Ex. 4. If the expressions $(px^3 + 3x^2 - 3)$ and $(2x^3 - 5x + p)$ when divided by $(x - 4)$ leave the same remainder, then what is the value of p ?

Sol. Let $f(x) = px^3 + 3x^2 - 3$

$g(x) = 2x^3 - 5x + p$

When divisible by $x - 4$, the remainders for the given expressions are $f(4)$ and $g(4)$ respectively.

$f(4) = p(4)^3 + 3(4)^2 - 3 = 64p + 48 - 3 = 64p + 45$

$g(4) = 2(4)^3 - 5(4) + p = 128 - 20 + p = 108 + p$.

Given, $f(4) = g(4) \Rightarrow 64p + 45 = 108 + p \Rightarrow 63p = 63 \Rightarrow p = \mathbf{1}$.

Ex. 5. What is/are the factors of $(x^{29} - x^{24} + x^{13} - 1)$?

(*a*) $(x-1)$ only **(*b*) $(x+1)$ only** **(*c*) $(x-1)$ and $(x+1)$** **(*d*) Neither $(x-1)$ nor $(x+1)$** ***(CDS 2008)***

Sol. For $(x - 1)$ to be a factor of the given expression, the value of expression at $x = 1$ is

$(1)^{29} - (1)^{24} + (1)^{13} - 1 = 1 - 1 + 1 - 1 = 0$

$\therefore (x - 1)$ is a factor of $x^{29} - x^{24} + x^{13} - 1$

Similarly for $(x + 1)$ to be the factor, the value of expression at $x = -1$ is

$(-1)^{29} - (-1)^{24} + (-1)^{13} - 1 = -1 - 1 - 1 - 1 = -4 \neq 0$

$\therefore (x + 1)$ is not a factor of $x^{29} - x^{24} + x^{13} - 1$.

Hence, (*a*) is the correct option.

Ex. 6. Which one of the following is one of the factors of $x^2(y-z) + y^2(z-x) - z(xy - yz - zx)$?

(a) $(x-y)$ **(b) $(x+y-z)$** **(c) $(x-y-z)$** **(d) $(x+y+z)$**

(CDS 2007)

Sol. $x^2(y-z) + y^2(z-x) - z(xy - yz - zx)$

$$= x^2y - x^2z + y^2z - y^2x - zxy + yz^2 + z^2x$$
$$= xy(x-y-z) + z^2(x+y) - z(x^2 - y^2)$$
$$= xy(x-y-z) - z(x+y)(x-y-z) = (x-y-z)(xy - yz - zx)$$

Hence, (*c*) is the correct option.

Ex. 7. Without actual division show that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

Sol. Let $f(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$ and $g(x) = x^2 - 3x + 2 = x^2 - 2x - x + 2 = x(x-2) - 1(x-2) = (x-2)(x-1)$

For $f(x)$ to be exactly divisible by $g(x)$, $(x-1)$ and $(x-2)$ should be the factors of $f(x)$, *i.e.*,

$$f(1) = 0 \text{ and } f(2) = 0.$$

Now, $f(1) = 2.(1)^4 - 6.(1)^3 + 3.(1)^2 + 3.1 - 2 = 2 - 6 + 3 + 3 - 2 = 0$

$f(2) = 2.(2)^4 - 6(2)^3 + 3(2)^2 + 3.2 - 2 = 32 - 48 + 12 + 6 - 2 = 0.$

$\therefore$ $(x-1)$ and $(x-2)$ are factors of $f(x) \Rightarrow f(x)$ is exactly divisible by $g(x)$.

Ex. 8. If $a + b + c = 0$, then what is the value of $a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2 - 2c^2a^2$? ***(CDS 2005)***

Sol. Given, $a + b + c = 0$.

Now, $a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2 - 2c^2a^2 = (a^2 + b^2 + c^2)^2 - 4a^2b^2 - 4b^2c^2 - 4c^2a^2$

$$= [(a+b+c)^2 - 2ab - 2bc - 2ca]^2 - 4a^2b^2 - 4b^2c^2 - 4c^2a^2$$
$$= [0^2 - 2ab - 2bc - 2ca]^2 - 4a^2b^2 - 4b^2c^2 - 4c^2a^2$$
$$= 4a^2b^2 + 4b^2c^2 + 4c^2a^2 + 8ab^2c + 8abc^2 + 8a^2bc - 4a^2b^2 - 4b^2c^2 - 4c^2a^2$$
$$= 8ab^2c + 8abc^2 + 8a^2bc = 8abc(b + c + a) = 8abc.0 = \mathbf{0}.$$

Ex. 9. If $x = \frac{a-b}{a+b}, y = \frac{b-c}{b+c}, z = \frac{c-a}{c+a}$, then what is the value of $\frac{1+x}{1-x}.\frac{1+y}{1-y}.\frac{1+z}{1-z}$? ***(CDS 2006)***

Sol. $x = \frac{a-b}{a+b} \Rightarrow \frac{1}{x} = \frac{a+b}{a-b}$

$\Rightarrow \frac{1+x}{1-x} = \frac{a+b+a-b}{a+b-a+b} = \frac{2a}{2b} \Rightarrow \frac{1+x}{1-x} = \frac{a}{b}$ (Applying componendo and dividendo)

Similarly, $\frac{1+y}{1-y} = \frac{b}{c}, \frac{1+z}{1-z} = \frac{c}{a}$ $\therefore$ $\frac{1+x}{1-x} \cdot \frac{1+y}{1-y} \cdot \frac{1+z}{1-z} = \frac{a}{b} \cdot \frac{b}{c} \cdot \frac{c}{a} = \mathbf{1}.$

Ex. 10. If $x + y + z = 0$, then what is $\left[\frac{(y-z-x)}{2}\right]^3 + \left[\frac{(z-x-y)}{2}\right]^3 + \left[\frac{(x-y-z)}{2}\right]^3$ equal to ?

Sol. $\left(\frac{y-z-x}{2}\right)^3 + \left(\frac{z-x-y}{2}\right)^3 + \left(\frac{x-y-z}{2}\right)^3$

$$= \left(\frac{y-(z+x)}{2}\right)^3 + \left(\frac{z-(x+y)}{2}\right)^3 + \left(\frac{x-(y+z)}{2}\right)^3$$

$$= \left(\frac{y-(-y)}{2}\right)^3 + \left(\frac{z-(-z)}{2}\right)^3 + \left(\frac{x-(-x)}{2}\right)^3 \qquad (\because x + y + z = 0)$$

$$= \left(\frac{2y}{2}\right)^3 + \left(\frac{2z}{2}\right)^3 + \left(\frac{2x}{2}\right)^3 = y^3 + z^3 + x^3 = \mathbf{3xyz}. \qquad (\because a^3 + b^3 + c^3 = 3abc, \text{ if } a + b + c = 0)$$

Ex. 11. If $x^2 - 4x + 1 = 0$, then what is the value of $x^3 + \frac{1}{x^3}$?

Sol. $x^2 - 4x + 1 = 0$

$$\Rightarrow x = \frac{4 \pm \sqrt{16 - 4\times1\times1}}{2\times1} = \frac{4\pm2\sqrt{3}}{2} = 2\pm\sqrt{3}$$

$\left[\text{Roots quadratic eqn } ax^2 + bx + c = 0 = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \text{ Here } a = 1, b = -4, c = 1\right]$

$$\therefore \quad x^3 + \frac{1}{x^3} = (2+\sqrt{3})^3 + \frac{1}{(2+\sqrt{3})^3} = (2+\sqrt{3})^3 + \left[\frac{(2-\sqrt{3})\times1}{(2+\sqrt{3})(2-\sqrt{3})}\right]^3 = (2+\sqrt{3})^3 + (2-\sqrt{3})^3$$

$$= 2^3 + (\sqrt{3})^3 + 3\times2\times\sqrt{3}(2+\sqrt{3}) + 2^3 - (\sqrt{3})^3 - 3\times2\times\sqrt{3}(2-\sqrt{3})$$

$= 8 + 18 + 8 + 18 =$ **52.** Similarly for $x = 2 - \sqrt{3}$, $x^3 + \frac{1}{x^3} =$ **52.**

Ex. 12. If $\frac{1}{y+z} + \frac{1}{z+x} = \frac{2}{x+y}$, then what is $(x^2 + y^2)$ equal to ?

Sol. $\frac{1}{y+z} + \frac{1}{z+x} = \frac{2}{x+y}$

$$\Rightarrow \frac{1}{y+z} - \frac{1}{x+y} = \frac{1}{x+y} - \frac{1}{z+x} \quad \Rightarrow \frac{(x+y)-(y+z)}{(y+z)(x+y)} = \frac{(z+x)-(x+y)}{(x+y)(z+x)}$$

$$\Rightarrow \frac{x-z}{y+z} = \frac{z-y}{z+x} \Rightarrow (x-z)(x+z) = (z-y)(z+y) \quad \Rightarrow x^2 - z^2 = z^2 - y^2 \Rightarrow x^2 + y^2 = \mathbf{2z^2}.$$

Ex. 13. If the sum and difference of two expressions are $5a^2 - a - 4$ and $a^2 + 9a - 10$ respectively, then what is their LCM ?

Sol. Let P and Q be the two expressions. Then,

$P + Q = 5a^2 - a - 4$ ...(*i*)

$P - Q = a^2 + 9a - 10$ ...(*ii*)

Adding (*i*) and (*ii*)

$\Rightarrow 2P = 6a^2 + 8a - 14 \Rightarrow P = 3a^2 + 4a - 7 = (a - 1)(3a + 7)$

From (*i*), $Q = (5a^2 - a - 4) - (3a^2 + 4a - 7) = 2a^2 - 5a + 3 = (a - 1)(2a - 3)$

$\therefore$ LCM of P and $Q = (a - 1)(2a - 3)(3a + 7)$.

Ex. 14. Without actual division, show that $(x - 1)^{2n} - x^{2n} + 2x - 1$ is divisible by $2x^3 - 3x^2 + x$.

Sol. Let $f(x) = (x-1)^{2n} - x^{2x} + 2x - 1$

$g(x) = 2x^3 - 3x^2 + x = x(2x^2 - 3x + 1)$

Now $g(x) = 0 \Rightarrow x[2x^2 - 3x + 1] = 0 \Rightarrow x[2x^2 - 2x - x + 1] = 0$

$\Rightarrow [2x(x-1) - 1(x-1)] = 0 \Rightarrow (2x-1)(x-1) = 0 \Rightarrow x = \frac{1}{2}, 1$

$\therefore$ For $f(x)$ to be exactly divisible by $g(x)$, $f\left(\frac{1}{2}\right) = f(1)$ should be all zero.

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}-1\right)^{2n} - \left(\frac{1}{2}\right)^{2n} + 2\times\frac{1}{2} - 1 = \left(-\frac{1}{2}\right)^{2n} - \left(\frac{1}{2}\right)^{2n} + 1 - 1 = \left(\frac{1}{2}\right)^{2n} - \left(\frac{1}{2}\right)^{2n} + 1 - 1 = 0$$

$f(1) = (1-1)^{2n} - 1^{2n} + 2\times1 - 1 = 0 - 1 + 2 - 1 = 0.$

$\therefore [(x-1)^{2n} - x^{2n} + 2x - 1]$ is completely divisible by $2x^3 - 3x^2 + x$.

Ex. 15. If the HCF of $(x^2 + x - 12)$ and $(2x^2 - kx - 9)$ is $(x - k)$, then what is the value of k ? ***(CDS 2008)***

Sol. Since $(x - k)$ is the HCF of $(x^2 + x + 12)$ and $(2x^2 - kx - 9)$

$(x - k)$ will be a factor of $2x^2 - kx - 9$

$\therefore 2.k^2 - k.k - 9 = 0 \Rightarrow k^2 - 9 = 0 \Rightarrow k = \pm 3$

Also, the factors of $(x^2 + x - 12) = (x + 4)(x - 3)$ $\therefore$ $\mathbf{k = 3.}$

PRACTICE SHEET

LEVEL–1

1. When $x^{13} + 1$ is divided by $x - 1$, the remainder is :

(*a*) 1 (*b*) –1 (*c*) 0 (*d*) 2

2. If $x^3 + 5x^2 + 10k$ leaves remainder $-2x$ when divided by $x^2 + 2$, then what is the value of k ?

(*a*) –2 (*b*) –1 (*c*) 1 (*d*) 2

(*CDS 2012*)

3. $x^4 + xy^3 + x^3y + xz^3 + y^4 + yz^3$ is divisible by :

(*a*) $(x - y)$ only (*b*) $(x^3 + y^3 + z^3)$ only

(*c*) both $(x + y)$ and $(x^3 + y^3 + z^3)$

(*d*) None of the above **(*CDS 2012*)**

4. For what value of k, will the expression $(3x^3 - kx^2 + 4x + 16)$ be divisible by $(x - k/2)$?

(*a*) 4 (*b*) –4 (*c*) 2 (*d*) 0

(*CDS 2007*)

5. When $(x^3 - 2x^2 + px - q)$ is divided by $(x^2 - 2x - 3)$, the remainder is $(x - 6)$, What are the values of p and q respectively ?

(*a*) –2, –6 (*b*) 2, – 6 (*c*) –2, 6 (*d*) 2, 6

(*CDS 2009*)

6. If $\dfrac{x^3 + ax^2 + bx + 4}{x^2 + x - 2}$ is a polynomial of degree 1 in x, then what are the values of a, b respectively ?

(*a*) –1, –4 (*b*) –1, 4 (*c*) 3, –4 (*d*) 3, 4

(*CDS 2005*)

7. When $a + b + c + 3a^{1/3}b^{2/3} + 3a^{2/3}b^{1/3}$ is divided by $a^{1/3} + b^{1/3} + c^{1/3}$, what is the remainder ?

(*a*) $3a$ (*b*) $3b$ (*c*) 0 (*d*) $c^{2/3}$

(*CDS 2005*)

8. If the polynomials $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ leave the same remainder when divided by $(x - 3)$, the value of a is :

(*a*) 2 (*b*) –3/2 (*c*) –1 (*d*) 4

9. Let R_1 and R_2 be the remainders when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^2 + ax^2 - 12x + 6$ are divided by $(x + 1)$ and $(x - 2)$ respectively. If $2R_1 + R_2 = 6$, the value of a is :

(*a*) –2 (*b*) 1 (*c*) –1 (*d*) 2

10. If both $(x - 2)$ and $(x - 1/2)$ are factors of $px^2 + 5x + r$, then:

(*a*) $p = 2r$ (*b*) $p + r = 0$ (*c*) $p = r$ (*d*) $p \times r = 1$

11. If the expression $ax^2 + bx + c$ is equal to 4, when $x = 0$, leaves a remainder 4 when divided by $x + 1$ and leaves a remainder 6 when divided by $x + 2$, then the values of a, b and c are respectively,

(*a*) 1, 1, 4 (*b*) 2, 2, 4 (*c*) 3, 3, 4 (*d*) 4, 4, 4

12. $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $(x - 1)$ and $(x + 1)$, the remainders are respectively 5 and 19. Determine the remainder when $f(x)$ is divided by $(x - 2)$.

(*a*) 6 (*b*) 10 (*c*) 2 (*d*) 8

13. If $(x^2 - 1)$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, then :

(*a*) $a + c + e = 0$ (*b*) $ace = 1$

(*c*) $b + d = 0$ (*d*) Both (*a*) and (*c*)

14. What is $\dfrac{x^2 - 3x + 2}{x^2 - 5x + 6} \div \dfrac{x^2 - 5x + 4}{x^2 - 7x + 12}$ equal to ?

(*a*) $\dfrac{x+3}{x-3}$ (*b*) 1 (*c*) $\dfrac{x+1}{x-1}$ (*d*) 2

(*CDS 2011*)

15. If the expression $(px^3 + x^2 - 2x - q)$ is divisible by $(x - 1)$ and $(x + 1)$, then the values of p and q respectively are ?

(*a*) 2, – 1 (*b*) –2, 1 (*c*) –2, –1 (*d*) 2, 1

(*CDS 2010*)

LEVEL–2

16. When $x^{40} + 2$ is divided by $x^4 + 1$, what is the remainder ?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

(*CDS 2009*)

17. If the remainder of the polynomial $a_0 + a_1x + a_2x^2 + ... + a_nx^n$ when divided by $(x - 1)$ is 1, then which one of the following is correct ?

(*a*) $a_0 + a_2 + ... = a_1 + a_3 + ...$

(*b*) $a_0 + a_2 + ... = 1 + a_1 + a_3 + ...$

(*c*) $1 + a_0 + a_2 + ... = -(a_1 + a_3 + ...)$

(*d*) $1 - a_0 - a_2 - ...... = a_1 + a_3 + ...$ **(*CDS 2009*)**

18. The remainder when $1 + x + x^2 + x^3 + ... + x^{1007}$ is divided by $(x - 1)$ is

(*a*) 1006 (*b*) 1008 (*c*) 1007 (*d*) 0

19. A cubic polynomial $f(x)$ is such that $f(1) = 1$, $f(2) = 2$, $f(3) = 3$ and $f(4) = 5$, then $f(6)$ equals :

(*a*) 7 (*b*) 6 (*c*) 10 (*d*) 13

20. If the polynomial $x^6 + px^5 + qx^4 - x^2 - x - 3$ is divisible by $x^4 - 1$, then the value of $p^2 + q^2$ is :

(*a*) 1 (*b*) 9 (*c*) 10 (*d*) 13

(*CDS 2001*)

21. The factors of $x^8 + x^4 + 1$ are :

(*a*) $(x^4 + 1 - x^2)(x^2 + 1 + x)(x^2 + 1 - x)$
(*b*) $(x^4 + 1 - x^2)(x^2 - 1 + x)(x^2 + 1 + x)$
(*c*) $(x^4 - 1 + x^2)(x^2 - 1 + x)(x^2 + 1 + x)$
(*d*) $(x^4 - 1 + x^2)(x^2 + 1 - x)(x^2 + 1 + x)$ **(*CDS 1999*)**

22. If the polynomial $x^{19} + x^{17} + x^{13} + x^{11} + x^7 + x^5 + x^3$ is divided by $(x^2 + 1)$, then the remainder is :

(*a*) 1 (*b*) $x^2 + 4$ (*c*) $-x$ (*d*) x

23. If $(x + k)$ is a common factor of $x^2 + px + q$ and $x^2 + lx + m$, then the value of k is

(*a*) $\frac{p+q}{l+m}$ (*b*) $\frac{p-l}{q-m}$ (*c*) $\frac{q+m}{q+l}$ (*d*) $\frac{q-m}{p-l}$

24. If $(x - 1)$ is a factor of $Ax^3 + Bx^2 - 36x + 22$ and $2^B = 64^A$, find A and B ?

(*a*) $A = 4, B = 16$ (*b*) $A = 6, B = 24$
(*c*) $A = 2, B = 12$ (*d*) $A = 8, B = 16$

25. When a polynomial $f(x)$ is divided by $(x - 3)$ and $(x + 6)$, the respective remainders are 7 and 22. What is the remainder when $f(x)$ is divided by $(x - 3)(x + 6)$?

(*a*) $\frac{-5}{3}x + 12$ (*b*) $-\frac{7}{3}x + 14$ (*c*) $-\frac{5}{3}x + 16$ (*d*) $-\frac{7}{3}x + 12$

26. If $p(x)$ is a common multiple of degree 6 of the polynomials $f(x) = x^3 + x^2 - x - 1$ and $g(x) = x^3 - x^2 + x - 1$, then which one of the following is correct ?

(*a*) $p(x) = (x - 1)^2 (x + 1)^2 (x^2 + 1)$
(*b*) $p(x) = (x - 1)(x + 1)(x^2 + 1)^2$
(*c*) $p(x) = (x - 1)^3 (x + 1)(x^2 + 1)$
(*d*) $p(x) = (x - 1)^2 (x^4 + 1)$ **(*CDS 2012*)**

27. Which one of the following is divisible by $(1 + a + a^5)$ and $(1 + a^4 + a^5)$ individually ?

(*a*) $(a^2 + a + 1)(a^3 + a^2 + 1)(a^3 + a + 1)$
(*b*) $(a^4 - a + 1)(a^3 + a^2 + 1)(a^3 + a - 1)$
(*c*) $(a^4 + a + 1)(a^3 - a^2 + 1)(a^3 + a + 1)$
(*d*) $(a^2 + a + 1)(a^3 - a^2 + 1)(a^3 - a + 1)$ **(*CDS 2005*)**

28. Consider the following statements :

1. $a^n + b^n$ is divisible by $a + b$ if $n = 2k + 1$, where k is a positive integer.
2. $a^n - b^n$ is divisible by $a - b$ if $n = 2k$, where k is a positive integer. Which of the statements given above is/are correct ?

(*a*) 1 only (*b*) 2 only
(*c*) Both 1 and 2 (*d*) Neither 1 nor 2 **(*CDS 2005*)**

29. If $(x - 2)$ is a common factor of the expressions $x^2 + ax + b$ and $x^2 + cx + d$, then $\frac{b-d}{c-a}$ is equal to

(*a*) –2 (*b*) –1 (*c*) 1 (*d*) 2 **(*EAMCET 2004*)**

30. Let $a \neq 0$ and $p(x)$ be a polynomial of degree greater than 2. If $p(x)$ leaves remainders a and $-a$, when divided respectively by $(x + a)$ and $(x - a)$, then the remainder when $p(x)$ is divided by $(x^2 - a^2)$ is

(*a*) $-2x$ (*b*) $-x$ (*c*) 0 (*d*) $2a$ **(*EAMCET 2003*)**

31. If $9x^2 + 3px + 6q$ when divided by $(3x + 1)$ leaves a remainder $\left(-\frac{3}{4}\right)$ and $qx^2 + 4px + 7$ is exactly divisible by $(x + 1)$, then the values of p and q respectively will be :

(*a*) $0, \frac{7}{4}$ (*b*) $-\frac{7}{4}, 0$ (*c*) Same (*d*) $\frac{7}{4}, 0$

32. What should be subtracted from $27x^3 - 9x^2 - 6x - 5$ to make it exactly divisible by $(3x - 1)$

(*a*) –5 (*b*) –7 (*c*) 5 (*d*) 7 **(*CDS 2009*)**

33. The values of a, b and c respectively for the expression $f(x) = x^3 + ax^2 + bx + c$, if $f(1) = f(2) = 0$ and $f(4) = f(0)$ are :

(*a*) 9, 20, 12 (*b*) –9, –20, 12
(*c*) –9, 20, –12 (*d*) –9, –20, –12

34. The remainder, when x^{200} is divided by $x^2 - 3x + 2$ is

(*a*) $(2^{200} - 1)x + (-2^{200} + 2)$
(*b*) $(2^{200} + 1)x + (-2^{200} - 2)$
(*c*) $(2^{200} - 1)x + (-2^{200} - 2)$
(*d*) 2^{100}

35. (*i*) For $a \neq b$, if $x + k$ is the HCF of $x^2 + ax + b$ and $x^2 + bx + a$, then the value of $a + b$ is equal to

(*a*) – 2 (*b*) – 1 (*c*) 0 (*d*) 2 **(*Type Raj PET 2004, Kerala PET 2004*)**

(*ii*) If $(x + k)$ is the HCF of $ax^2 + ax + b$ and $x^2 + cx + d$, then what is the value of k ?

(*a*) $\frac{b+d}{a+c}$ (*b*) $\frac{a+b}{c+d}$ (*c*) $\frac{a-b}{c-d}$

(*d*) None of these **(*CDS 2008*)**

LEVEL–3

36. The value of $\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$ is :

(*a*) 1 (*b*) 3 (*c*) 1/3 (*d*) Zero **(*CDS 2000*)**

37. If $a^2 = by + cz$, $b^2 = cz + ax$, $c^2 = ax + by$, then the value of $\frac{x}{a+x} + \frac{y}{b+y} + \frac{z}{c+z}$ will be:

(*a*) $a + b + c$ (*b*) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ (*c*) 1 (*d*) 0 **(*CDS 2001*)**

38. If $x + y + z = 0$, then $x(y - z)^3 + y(z - x)^3 + z(x - y)^3$ equals

(*a*) 0 (*b*) $y + z$ (*c*) 1 (*d*) $(z + x)^2$

39. If $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{(a+b+c)}$, where $a + b + c \neq 0$, $abc \neq 0$, what is the value of $(a + b)(b + c)(c + a)$?

(*a*) 0 (*b*) 1 (*c*) –1 (*d*) 2 **(*CDS 2005*)**

40. If $\dfrac{x}{(b-c)(b+c-2a)} = \dfrac{y}{(c-a)(c+a-2b)}$ $= \dfrac{z}{(a-b)(a+b-2c)}$, what is the value of $x + y + z$?

(*a*) $(a + b + c)$ (*b*) $a^2 + b^2 + c^2$
(*c*) 0 (*d*) 1 **(*CDS 2005*)**

41. If $a + b + c = 0$, then find the value of :

$\dfrac{a^2}{a^2-bc} + \dfrac{b^2}{b^2-ca} + \dfrac{c^2}{c^2-ab}$

(*a*) 4 (*b*) 2 (*c*) 1 (*d*) 0
(*MAT 2005*)

42. The value of $\dfrac{(x-y)^3+(y-z)^3+(z-x)^3}{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}$ is :

(*a*) 1 (*b*) $[2(x + y + z)]^{-1}$
(*c*) $[(x + y)(y + z)(z + x)]^{-1}$ (*d*) 0 **(*CDS 2004*)**

43. If $a + b + c = 0$, then $a^2 + ab + b^2$ is equal to :

(*a*) $b^2 - bc + c^2$ (*b*) $c^2 - ab$
(*c*) $b^2 + bc + c^2$ (*d*) 0 **(*CDS 2004*)**

44. If $pqr = 1$, the value of $\dfrac{1}{[1+p+q^{-1}]} + \dfrac{1}{[1+q+r^{-1}]}$ $+ \dfrac{1}{[1+r+p^{-1}]}$ will be equal to :

(*a*) 1 (*b*) 0 (*c*) –1 (*d*) –2
(*CDS 2004*)

45. If $a = \dfrac{xy}{x+y}$, $b = \dfrac{xz}{x+z}$ and $c = \dfrac{yz}{y+z}$, where *a, b* and *c* are non-zero, then what is *x* equal to ?

(*a*) $\dfrac{2abc}{ac+bc-ab}$ (*b*) $\dfrac{2abc}{ab-ac+bc}$

(*c*) $\dfrac{2abc}{ab+bc+ac}$ (*d*) $\dfrac{2abc}{ab+ac-bc}$
(*CDS 2006*)

46. If $a + b + c = 0$, then what is the value of

$\dfrac{a^2+b^2+c^2}{(a-b)^2+(b-c)^2+(c-a)^2}$

(*a*) 1 (*b*) 3 (*c*) $\dfrac{1}{3}$ (*d*) 0 **(*CDS 2006*)**

47. If $x + y + z = 2s$, then what is

$(s - x)^3 + (s - y)^3 + 3(s - x)(s - y)z$ equal to :

(*a*) z^3 (*b*) $-z^3$ (*c*) x^3 (*d*) y^3
(*CDS 2007*)

48. If $x + \dfrac{1}{x} = p$, then $x^6 + \dfrac{1}{x^6}$ equals to :

(*a*) $p^6 + 6p$ (*b*) $p^6 - 6p$
(*c*) $p^6 + 6p^4 + 9p^2 + 2$ (*d*) $p^6 - 6p^4 + 9p^2 - 2$
(*CDS 2007*)

49. If $x + y + z = 0$, then what is the value of :

$\dfrac{1}{x^2+y^2-z^2} + \dfrac{1}{y^2+z^2-x^2} + \dfrac{1}{z^2+x^2-y^2}$?

(*a*) $\dfrac{1}{x^2+y^2+z^2}$ (*b*) 1
(*c*) –1 (*d*) 0 **(*CDS 2010*)**

50. If $x + y + z = 0$, then $\dfrac{x^2}{2x^2+yz} + \dfrac{y^2}{2y^2+zx} + \dfrac{z^2}{2z^2+xy} =$

(*a*) 4 (*b*) 2 (*c*) 3 (*d*) 1

51. If $(b + c - a)x = (c + a - b)y = (a + b - c)z = 2$, then $\left(\dfrac{1}{y}+\dfrac{1}{z}\right)\left(\dfrac{1}{z}+\dfrac{1}{x}\right)\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$ is equals:

(*a*) $a^2b^2c^2$ (*b*) abc (*c*) a^2b^2 (*d*) $(abc)^2$

52. If $a^x = (x + y + z)^y$, $a^y = (x + y + z)^z$, $a^z = (x + y + z)^x$, then:

(*a*) $3(x + y + z) = a$ (*b*) $2a = x + y + z$
(*c*) $x + y + z = 0$ (*d*) $x = y = z = a/3$

53. If $x + \dfrac{1}{x} = a$, then what is the value of $x^3 + x^2 + \dfrac{1}{x^3} + \dfrac{1}{x^2}$?

(*a*) $a^3 + a^2$ (*b*) $a^3 + a^2 - 5a$
(*c*) $a^3 + a^2 - 3a - 2$ (*d*) $a^3 + a^2 - 4a - 2$
(*CDS 2012*)

54. If $x^{1/3} + y^{1/3} + z^{1/3} = 0$, then what is $(x + y + z)^3$ equal to ?

(*a*) 1 (*b*) 3 (*c*) $3xy$ (*d*) $27xyz$

55. $\dfrac{2a}{a+b} + \dfrac{2b}{b+c} + \dfrac{2c}{c+a} + \dfrac{(b-c)(c-a)(a-b)}{(b+c)(c+a)(a+b)}$ equals

(*a*) 0 (*b*) –1 (*c*) 3 (*d*) 2

ANSWERS

1. (*d*)	**2.** (*c*)	**3.** (*b*)	**4.** (*b*)	**5.** (*c*)	**6.** (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*d*)	**10.** (*c*)
11. (*a*)	**12.** (*b*)	**13.** (*d*)	**14.** (*b*)	**15.** (*d*)	**16.** (*c*)	**17.** (*d*)	**18.** (*b*)	**19.** (*b*)	**20.** (*c*)
21. (*a*)	**22.** (*c*)	**23.** (*d*)	**24.** (*c*)	**25.** (*a*)	**26.** (*a*)	**27.** (*b*)	**28.** (*c*)	**29.** (*d*)	**30.** (*b*)
31. (*d*)	**32.** (*b*)	**33.** (*c*)	**34.** (*a*)	**35.** (*i*) (*b*) (*ii*) (*d*)		**36.** (*b*)	**37.** (*c*)	**38.** (*a*)	**39.** (*a*)
40. (*c*)	**41.** (*b*)	**42.** (*c*)	**43.** (*b*)	**44.** (*a*)	**45.** (*a*)	**46.** (*c*)	**47.** (*a*)	**48.** (*d*)	**49.** (*d*)
50. (*d*)	**51.** (*b*)	**52.** (*d*)	**53.** (*c*)	**54.** (*d*)	**55.** (*c*)				

HINTS AND SOLUTIONS

1. Remainder when $x^{13} + 1$ is divided by $(x - 1) = 1^{13} + 1 = \mathbf{2.}$

2.
$$\begin{array}{r|l} & x + 5 \\ \hline x^2+2 & x^3 + 5x^2 \qquad + 10k \\ & \underline{\overset{-}{x^3} \qquad \overset{-}{+} 2x} \\ & 5x^2 - 2x + 10k \\ & \underline{\overset{-}{5x^2} \qquad \overset{-}{+} 10} \\ & -2x + 10k - 10 \quad = \text{Remainder} \end{array}$$

Given, $-2x + 10k - 10 = -2x$

$\Rightarrow 10k = 10 \Rightarrow k = \mathbf{1.}$

3. Hint. $x^4 + xy^3 + x^3y + xz^3 + y^4 + yz^3$

$= (x^4 + xy^3 + xz^3) + (x^3y + y^4 + yz^3)$

$= x(x^3 + y^3 + z^3) + y(x^3 + y^3 + z^3)$

$= (x + y)(x^3 + y^3 + z^3)$

4. Let $f(x) = 3x^3 - kx^2 + 4x + 16$. Then, $f(x)$ will be divisible by $(x - k/2)$ if $f(k/2) = 0$

$\Rightarrow 3.(k/2)^3 - k.(k/2)^2 + 4(k/2) + 16 = 0$

$\Rightarrow \dfrac{3k^3}{8} - \dfrac{k^3}{4} + \dfrac{4k}{2} + 16 = 0$

$\Rightarrow \dfrac{3k^3 - 2k^3 + 16k + 128}{8} = 0$

$\Rightarrow k^3 + 16k + 128 = 0 \Rightarrow (k + 4)(k^2 - 4k + 32) = 0$

$\Rightarrow k + 4 = 0 \Rightarrow k = \mathbf{-4.}$

5.
$$\begin{array}{r|l} & x \\ \hline x^2 - 2x - 3 & x^3 - 2x^2 + px - q \\ & \underline{x^3 \overset{-}{\underset{+}{}} 2x^2 \overset{-}{\underset{+}{}} 3x} \\ & (p + 3)x - q \end{array}$$

Given, $(p + 3)x - q = x - 6$

$\Rightarrow p + 3 = 1$ and $q = 6$

$\Rightarrow p = \mathbf{-2}$, $q = \mathbf{6}$

6.
$$\begin{array}{r|l} & x + (a - 1) \\ \hline x^2 + x - 2 & x^3 + ax^2 + bx + 4 \\ & \underline{\overset{-}{x^3} \overset{-}{+} x^2 \overset{+}{-} 2x} \\ & (a - 1)x^2 + (b + 2)x + 4 \\ & \underline{\overset{-}{(a - 1)x^2} \overset{-}{+} (a - 1)x \overset{+}{-} 2(a - 1)} \\ & (b - a + 3)x + (2a + 2) \end{array}$$

As the given polynomial is of degree 1, the degree of the remainder should be less than 1, *i.e.*, 0, *i.e.*, the remainder has only a constant term.

$\Rightarrow b - a + 3 = 0$ and $2a + 2 = 0 \Rightarrow a = -1$

$\therefore b - (-1) + 3 = 0 \Rightarrow b = -4.$

$\therefore \boldsymbol{a = -1, b = -4.}$

7. Let $a^{1/3} = x$, $b^{1/3} = y$, $c^{1/3} = z$. Then,

$a + b + c + 3a^{1/3}\, b^{2/3} + 3a^{2/3}\, b^{1/3} = x^3 + y^3 + z^3 + 3xy^2 + 3x^2y$

and $a^{1/3} + b^{1/3} + c^{1/3} = x + y + z$.

Now $x^3 + y^3 + z^3 + 3xy^2 + 3x^2y$

$= x^3 + 3xy^2 + 3x^2y + y^3 + z^3$

$= (x + y)^3 + z^3$

$= [x + y + z][(x + y)^2 - (x + y)z + z^2].$

$\therefore$ Given expression is completely divisible by $(x + y + z)$, *i.e.*, by $a^{1/3} + b^{1/3} + c^{1/3}$.

8. Let $f(x) = ax^3 + 4x^2 + 3x - 4$

$g(x) = x^3 - 4x + a.$

Remainders when $f(x)$ and $g(x)$ are divided by $(x - 3)$ are $f(3)$ and $g(3)$ respectively. Now,

$f(3) = a.(3)^3 + 4.(3)^2 + 3.3 - 4$

$= 27a + 36 + 9 - 4 = 27a + 41$ ...(*i*)

$g(3) = (3)^3 - 4.(3) + a = 27 - 12 + a = 15 + a$ ...(*ii*)

Given, $f(3) = g(3)$

$\therefore 27a + 41 = 15 + a \Rightarrow 26a = -26 \Rightarrow a = \mathbf{-1.}$

9. Let $f(x) = x^3 + 2x^2 - 5ax - 7$

$\therefore \quad R_1 = f(-1) = (-1)^3 + 2.(-1)^2 - 5.a.(-1) - 7$

$= -1 + 2 + 5a - 7 = 5a - 6$

$g(x) = x^3 + ax^2 - 12x + 6$

$R_2 = g(2) = (2)^3 + a.(2)^2 - 12.(2) + 6$

$= 8 + 4a - 24 + 6 = 4a - 10$

Given, $2R_1 + R_2 = 6 \Rightarrow 2(5a - 6) + (4a - 10) = 6$

$\Rightarrow \quad 10a - 12 + 4a - 10 = 6$

$\Rightarrow \quad 14a - 22 = 6 \Rightarrow 14a = 28 \Rightarrow a = \mathbf{2.}$

10. Let $f(x) = px^2 + 5x + r$

Since, $(x - 2)$ and $\left(x - \dfrac{1}{2}\right)$ are the factors of $f(x)$, therefore,

$f(2) = 0$ and $f\left(\dfrac{1}{2}\right) = 0.$

$\therefore f(2) = p \times (2)^2 + 5 \times 2 + r = 4p + 10 + r = 0$ ...(*i*)

$f\left(\dfrac{1}{2}\right) = p \times \left(\dfrac{1}{2}\right)^2 + 5 \times \dfrac{1}{2} + r = \dfrac{p}{4} + \dfrac{5}{2} + r = p + 10 + 4r = 0$ ...(*ii*)

(*i*) and (*ii*) $\Rightarrow 4p + 10 + r = p + 10 + 4r \Rightarrow 3p = 3r \Rightarrow \boldsymbol{p = r.}$

11. Given exp. $f(x) = ax^2 + bx + c$

$\therefore$ When $x = 0$, $a.0 + b.0 + c = 4 \Rightarrow \mathbf{c = 4.}$

The remainders when $f(x)$ is divided by $(x + 1)$ and $(x + 2)$ respectively are $f(-1)$ and $f(-2)$.

$\therefore \quad f(-1) = a.(-1)^2 + b.(-1) + c = 4$

$\Rightarrow \quad a - b + c = 4 \Rightarrow a - b + 4 = 4 \Rightarrow a - b = 0$ ...(*i*)

$f(-2) = a.(-2)^2 + b(-2) + c = 6$

$\Rightarrow \quad 4a - 2b + 4 = 6 \Rightarrow 4a - 2b = 2$ ...(*ii*)

Solving (*i*) and (*ii*) simultaneously we get, $\boldsymbol{a = 1, b = 1.}$

12. When $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $(x - 1)$ and $(x + 1)$, the remainders are 5 and 19 respectively.

i.e., $f(1) = 5$ and $f(-1) = 19$

$\Rightarrow 1 - 2 + 3 - a + b = 5$ and $1 + 2 + 3 + a + b = 19$

$\Rightarrow -a + b = 3$ and $a + b = 13$

Adding the two equations, we get $2b = 16 \Rightarrow b = 8 \Rightarrow a = 5$

$\therefore f(x) = x^4 - 2x^3 + 3x^2 - ax + b$

$= x^4 - 2x^3 + 3x^2 - 5x + 8$

$\therefore$ Remainder, when $f(x)$ is divided by $(x - 2)$ is equal to $f(2)$

$\therefore f(2) = 2^4 - 2.2^3 + 3.2^2 - 5.2 + 8$

$= 16 - 16 + 12 - 10 + 8 =$ **10.**

13. Let $f(x) = ax^4 + bx^4 + cx^2 + dx + e$ be the given polynomial. Then, $(x^2 - 1)$ is a factor of $f(x)$.

$\Rightarrow (x - 1)(x + 1)$ is a factor of $f(x)$

$\Rightarrow (x - 1)$ and $(x + 1)$ are factors of $f(x)$

$\Rightarrow f(1) = 0$ and $f(-1) = 0$

$\Rightarrow a + b + c + d + e = 0$ and $a - b + c - d + e = 0$.

Adding and subtracting the two equations, we get

$2(a + c + e) = 0$ and $2(b + d) = 0$

$\Rightarrow a + c + e = 0$ and $b + d = 0$.

14. $\dfrac{x^2 - 3x + 2}{x^2 - 5x + 6} \div \dfrac{x^2 - 5x + 4}{x^2 - 7x + 12}$

$= \dfrac{(x-1)(x-2)}{(x-3)(x-2)} \div \dfrac{(x-4)(x-1)}{(x-3)(x-4)}$

$= \dfrac{(x-1)}{(x-3)} \div \dfrac{(x-1)}{(x-3)} = \dfrac{(x-1)}{(x-3)} \times \dfrac{(x-3)}{(x-1)} = \mathbf{1.}$

15. $px^3 + x^2 - 2x - q$ is divisible by $(x - 1)$ and $(x + 1)$

$\Rightarrow p(1)^3 + (1)^2 - 2(1) - q = 0 \Rightarrow p - q = 1$ ...(*i*)

and $p(-1)^3 + (-1)^2 - 2(-1) - q = 0 \Rightarrow p + q = 3$ ...(*ii*)

Solving (*i*) and (*ii*) $p = 2, q = 1$.

16. Put $x^4 = -1$ in $f(x) = x^{40} + 2$

Remainder $= (x^4)^{10} + 2 = (-1)^{10} + 2 =$ **3.**

17. Let $f(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots + a_nx^n$

Given, $f(1) = 1$

$\Rightarrow a_0 + a_1 + a_2 + \ldots\ldots\ldots + a_n = 1$

$\Rightarrow 1 - a_0 - a_2 - \ldots\ldots\ldots = a_1 + a_3 + \ldots\ldots\ldots$.

18. Required remainder $= f(1)$

$= 1 + 1 + 1 + 1 + 1 \ldots\ldots\ldots + 1$ (1008 times)

$= 1008 \times 1 =$ **1008.**

19. Let the cubic polynomial be :

$f(x) = ax^3 + bx^2 + cx + d$.

Given, $f(1) = 1 \Rightarrow a + b + c + d = 1$ ....(*i*)

$f(2) = 2 \Rightarrow 8a + 4b + 2c + d = 2$ ...(*ii*)

$f(3) = 4 \Rightarrow 27a + 9b + 3c + d = 3$ ...(*iii*)

$f(4) = 5 \Rightarrow 125a + 25b + 5c + d = 5$ ...(*iv*)

(*ii*) – (*i*) $\Rightarrow 7a + 3b + c = 1$ ...(*v*)

(*iii*) – (*ii*) $\Rightarrow 19a + 5b + c = 1$ ...(*vi*)

(*iv*) – (*iii*) $\Rightarrow 98a + 16b + 2c = 2$ ...(*vii*)

(*vi*) – (*v*) $\Rightarrow 12a + 2b = 0 \Rightarrow 6a + b = 0$ ...(*viii*)

(*vii*) – 2 (*vi*) $\Rightarrow 60a + 6b = 0 \Rightarrow 10a + b = 0$ ...(*ix*)

Solving (*viii*) and (*ix*), we get $a = 0 \Rightarrow b = 0$

Putting $a = 0, b = 0$ in (*v*), we, get $c = 1$

Also from (*i*), $a = 0, b = 0, c = 1 \Rightarrow d = 0$.

Putting values of a, b, c, d in $f(x) = ax^3 + bx^2 + cx + d$, we get the polynomial $f(x) = x \Rightarrow f(6) =$ **6.**

20. $f(x) = x^6 + px^5 + qx^4 - x^2 - x - 3$

$= x^4 . x^2 + p.x^4 x + q.x^4 - x^2 - x - 3$

As $(x^4 - 1)$ is a factor of $f(x)$, so putting $x^4 = 1$, we get

$x^2 + px + q - x^2 - x - 3 = 0$

$\Rightarrow (p - 1)x + (q - 3) = 0 \Rightarrow p - 1 = 0$ and $q - 3 = 0$

$\Rightarrow p = 1$ and $q = 3$.

$\therefore p^2 + q^2 = 1 + 9 =$ **10.**

21. $x^8 + x^4 + 1 = x^8 + 2x^4 + 1 - x^4$ (Adding and subtracting x^4)

$= (x^4 + 1)^2 - (x^2)^2 = (x^4 + 1 + x^2)(x^4 + 1 - x^2)$

$= [(x^4 + 2x^2 + 1) - x^2](x^4 + 1 - x^2)$

$= [(x^2 + 1)^2 - (x)^2](x^4 + 1 - x^2)$

$= (x^2 + 1 + x)(x^2 + 1 - x)(x^4 + 1 - x^2)$

22. $f(x) = x^{19} + x^{17} + x^{13} + x^{11} + x^7 + x^5 + x^3$

Putting $x^2 = -1$, we get

$f(x) = (x^2)^9.x + (x^2)^8.x + (x^2)^6.x + (x^2)^5.x + (x^2)^2.x + x^2.x$

$= (-1)^9x + (-1)^8.x + (-1)^6.x + (-1)^5.x + (-1)^2.x + (-1).x$

$= -x + x + x - x + x - x = \mathbf{-x.}$

23. Let $f(x) = x^2 + px + q$

$g(x) = x^2 + lx + m$.

Since $(x + k)$ is a common factor of $f(x)$ and $g(x)$,

$f(-k) = k^2 - pk + q = 0$

$g(-k) = k^2 - lk + m = 0$

$\Rightarrow k^2 - px + q = k^2 - lk + m$

$\Rightarrow q - m = (p - l)k \Rightarrow k = \dfrac{q - m}{p - l}$

24. Since $(x - 1)$ is a factor of $Ax^3 + Bx^2 - 36x + 22$

$\therefore A(1)^3 + B(1)^2 - 36(1) + 22 = 0$

$\Rightarrow A + B = 14$ ...(*i*)

and $2^B = 64^A \Rightarrow 2^B = (2^6)^A \Rightarrow B = 6A$ ...(*ii*)

$\therefore$ From (*i*) and (*ii*) $A = 2, B = 12$.

25. The function $f(x)$ is not known. Here,

$a = 3, \quad b = -6$

$A = 7, \quad B = 22$ **[*Refer to Key Fact 8*]**

$\therefore$ Required remainder $= \dfrac{A - B}{a - b}x + \dfrac{Ba - Ab}{a - b}$

$= \dfrac{7 - 22}{3 - (-6)}x + \dfrac{22 \times 3 - 7 \times (-6)}{3 - (-6)} = \mathbf{\dfrac{-5}{3}x + 12}$ **.**

26. $f(x) = x^3 + x^2 - x - 1$

$g(x) = x^3 - x^2 + x - 1$

$f(x) . g(x) = (x^3 + x^2 - x - 1) . (x^3 - x^2 + x - 1)$

$= x^6 - \cancel{x^5} + \cancel{x^4} - \cancel{x^3} + \cancel{x^5} - \cancel{x^4} + \cancel{x^3} - \cancel{x^2}$

$-x^4 + \cancel{x^3} - \cancel{x^2} + \cancel{x} - \cancel{x^3} + x^2 - \cancel{x} + 1$

$= x^6 - x^4 - x^2 + 1$

$\therefore p(x) = x^6 - x^4 - x^2 + 1$

$= x^4(x^2 - 1) - (x^2 - 1) = (x^2 - 1)(x^4 - 1)$

$= (x - 1)(x + 1)[(x^2)^2 - 1]$

$= (x - 1)(x + 1)[(x^2 - 1)(x^2 + 1)]$

$= (x-1)(x+1)[(x-1)(x+1)(x^2+1)]$

$= (x-1)^2(x+1)^2(x^2+1)$.

27. The given expression has to be divided by $(1 + a + a^5)$ and $(1 + a^4 + a^5)$ individually, so highest power of a is $5 + 5 = 10$, followed by $5 + 4 = 9$, and both are positive.

In options (a) and (d) the highest power of a is 8, hence these options are not acceptable.

The only choices are (b) and (c), but in option (c) a^{10} is positive but a^9 is negative and in (b) both a^{10} and a^9 are positive. Hence (b) is the correct option.

28. Statement (1) is correct as for $k = 1$, $n = 2 \times 1 + 1 = 3$.

$\therefore a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$ which is divisible by $(a + b)$, statement (2) is also correct as for $k = 1$, $n = 2$,

$\therefore a^2 - b^2 = (a - b)(a + b)$ which is divisible by $(a - b)$.

29. $(x - 2)$ is a common factor of $(x^2 + ax + b)$ and $(x^2 + cx + d)$

$\Rightarrow 4 + 2a + b = 0$ ...(i)

and $4 + 2c + d = 0$ ...(ii)

$\therefore 2a + b = 2c + d \Rightarrow b - d = 2(c - a) \Rightarrow \dfrac{b-d}{c-a} = \mathbf{2.}$

30. Let $rx + t$ be the remainder, $q(x)$ be the quotient when $p(x)$ is divided by $x^2 - a^2$.

$\therefore p(x) = (x^2 - a^2).\ qx + rx + t$ ...(i)

Given, $p(x)$ leaves remainders a and $-a$ respectively when divided by $(x + a)$ and $(x - a)$.

$\therefore p(-a) = a$ and $p(a) = -a$

Putting $x = -a$ in (i), we get

$p(-a) = 0.\ q(-a) + (-ra + t)$

$\Rightarrow a = -ra + t$ ...(ii)

Putting $x = a$, in (i), we get

$p(a) = 0.q\ (a) + (ra + t)$

$\Rightarrow -a = ra + t$ ...(iii)

$\therefore$ Adding (ii) and (iii), we get $2t = 0 \Rightarrow t = 0 \Rightarrow r = -1$

$\therefore$ Required remainder $= rx + t = \mathbf{-x.}$

31. Given, $(9x^2 + 3px + 6q)$, when divided by $(3x + 1)$ leaves a remainder $-\dfrac{3}{4}$

$\therefore f(x) = 9x^2 + 3px + 6q - \left(-\dfrac{3}{4}\right) = \left(9x^2 + 3px + 6q + \dfrac{3}{4}\right)$

is exactly divisible by $(3x + 1)$

$\therefore\ f\left(-\dfrac{1}{3}\right) = 0 \Rightarrow 9\left(-\dfrac{1}{3}\right)^2 + 3p.\left(-\dfrac{1}{3}\right) + 6q + \dfrac{3}{4} = 0$

$\Rightarrow 6q - p + \dfrac{7}{4} = 0$

$\Rightarrow 24q - 4p + 7 = 0$ ...(i)

Now, the expression $g(x) = qx^2 + 4px + 7$ is exactly divisible by $x + 1$

$\Rightarrow g(-1) = 0 \Rightarrow q - 4p + 7 = 0$ ...(ii)

Solving equations (i) and (ii), we get $q = \mathbf{0}$, $p = \mathbf{\dfrac{7}{4}}$.

32. To make $f(x) = 27x^3 - 9x^2 - 6x - 5$ exactly divisible by $(3x - 1)$, the remainder obtained on division should be subtracted.

Remainder $= f\left(+\dfrac{1}{3}\right) = 27 \times \left(\dfrac{1}{3}\right)^3 - 9 \times \left(\dfrac{1}{3}\right)^2 - 6 \times \dfrac{1}{3} - 5$

$\left(\because 3x - 1 = 0 \Rightarrow x = \dfrac{1}{3}\right)$

$= 1 - 1 - 2 - 5 = \mathbf{-7.}$

33. Given, $f(x) = x^3 + lx^2 + mx + n$.

$f(1) = f(2) = 0 \Rightarrow (x - 1)$ and $(x - 2)$ are factors of $f(x)$.

Since, $f(x)$ is polynomial of degree 3, it shall have three linear factors. So, let the third factor be $(x - k)$.

Then, $f(x) = (x - 1)(x - 2)(x - k)$

$\Rightarrow f(x) = x^3 + lx^2 + mx + n = (x - 1)(x - 2)(x - k)$

Given, $f(4) = f(0)$

$\Rightarrow (4 - 1)(4 - 2)(4 - k) = (-1)(-2)(-k)$

$\Rightarrow 24 - 6k = -2k \Rightarrow 4k = 24 \Rightarrow k = 6$

$\therefore f(x) = (x - 1)(x - 2)(x - 6) = (x^2 - 3x + 2)(x - 6)$

$= x^3 - 9x^2 + 20x - 12$

$\therefore x^3 + lx^2 + mx + n = x^3 - 9x^2 + 20x - 12$

$\Rightarrow l = -9$, $m = 20$, $n = -12$.

34. Let $x^{200} = (x^2 - 3x + 2).\ Q(x) + lx + m$ ...(i)

where, $Q(x)$ = quotient and $(lx + m)$ is the remainder

Now $(x^2 - 3x + 2) = 0 \Rightarrow (x - 1)\ (x - 2) = 0 \Rightarrow x = 1, 2$.

Substituting $x = 1$ in (i), we have,

$1^{200} = 0.\ Q.(x) + l + m$ ...(ii)

Similarly, for $x = 2$,

$2^{200} = 0.\ Q(x) + 2l + m$ ...(iii)

$\therefore l + m = 1,\ 2l + m = 2^{200}$

Solving we get, $l = 2^{200} - 1$ and $m = 2 - 2^{200}$

Hence remainder $= lx + m = (2^{200} - 1)\ x + (-2^{200} + 2)$.

35. (i) Since $x + k$ is the HCF of the given expressions, therefore, $x = -k$ will make each expression zero.

$k^2 - ak + b = 0$ ...(i)

$k^2 - bk + a = 0$ ...(ii)

Solving (i) and (ii) by the rule of cross multiplication,

$$\frac{k^2}{-a^2 + b^2} = \frac{k}{b - a} = \frac{1}{-b + a}$$

From last two relations, $k = \dfrac{b - a}{-(b - a)} = -1$

$\therefore\ \dfrac{k^2}{-a^2 + b^2} = \dfrac{1}{-b + a} \Rightarrow \dfrac{(-1)^2}{-a^2 + b^2} = \dfrac{1}{-b + a}$

$\Rightarrow \dfrac{1}{(b - a)(b + a)} = \dfrac{-1}{(b - a)} \Rightarrow a + b = -1.$

(ii) **Hint.** $ak^2 - ak + b = 0$

$k^2 - ck + d = 0$

Solving by the rule of cross multiplication,

$$\frac{k^2}{-ad + bc} = \frac{k}{b - ad} = \frac{1}{-ac + a}$$

$\Rightarrow k = \dfrac{b - ad}{a(1 - c)}, \dfrac{bc - ad}{b - ad}$.

36. Since $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$.

So, as $(a - b) + (b - c) + (c - a) = 0$

$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$

$\therefore$ Given expression $= \dfrac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = \mathbf{3.}$

37. $a^2 = by + cz \Rightarrow a^2 + ax = ax + by + cz$

$\Rightarrow a(a + x) = ax + by + cz$ ...(*i*)

Similarly, $b^2 = cz + ax \Rightarrow b(b + y) = ax + by + cz$ ...(*ii*)

and $c^2 = ax + by \Rightarrow c(c + z) = ax + by + cz$ ...(*iii*)

Hence, $\dfrac{x}{a+x} + \dfrac{y}{b+y} + \dfrac{c}{c+z}$

$= \dfrac{ax}{a(a+x)} + \dfrac{by}{b(b+y)} + \dfrac{cz}{c(c+z)}$

$= \dfrac{x.a}{ax+by+cz} + \dfrac{y.b}{ax+by+cz} + \dfrac{z.c}{ax+by+cz}$

$= \dfrac{ax+by+cz}{ax+by+cz} = \mathbf{1.}$

38. Now, $x + y + z = 0$

$\Rightarrow x = -y - z,\ y = -x - z,\ z = -x - y$

$\therefore x(y - z)^3 + y(z - x)^3 + z(x - y)^3$

$= (-y - z)(y - z)^3 + (-z - x)(z - x)^3 + (-x - y)(x - y)^3$

$= -(y + z)(y - z)^3 - (z + x)(z - x)^3 - (x + y)(x - y)^3$

$= -[(y^2 - z^2)(y - z)^2 + (z^2 - x^2)(z - x)^2 + (x^2 - y^2)(x - y)^2]$

$= -[(y^2 - z^2)(y^2 - 2yz + z^2) + (z^2 - x^2)(z^2 - 2xz + x^2)$
$+ (x^2 - y^2)(x^2 - 2xy + y^2)]$

$= -[(\cancel{y^4} - \cancel{z^4}) - 2yz(y^2 - z^2) + (\cancel{z^4} - \cancel{x^4})$
$- 2xz(z^2 - x^2) + (\cancel{x^4} - \cancel{y^4}) - 2xy(x^2 - y^2)]$

$= 2yz(y^2 - z^2) + 2xz(z^2 - x^2) + 2xy(x^2 - y^2)$

$= 2(y^3z - yz^3 + z^3x - x^3z + x^3y - xy^3)$

$= 2[x^3(y - z) + y^3(z - x) + z^3(x - y)]$

$= 2[-(y - z)(z - x)(x - y).(x + y + z)]$

$= \mathbf{0.}$ $(\because x + y + z = 0)$

39. $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = \dfrac{1}{a+b+c}$

$\Rightarrow (a + b + c)\left[\dfrac{bc + ac + ab}{abc}\right] = 1$

$\Rightarrow (a + b + c)(bc + ac + ab) = abc$

$\Rightarrow abc + a^2c + a^2b + b^2c + abc + ab^2 + bc^2 + ac^2 + abc = abc$

$\Rightarrow a^2(c + b) + bc(c + b) + ab(c + b) + ac(c + b) = 0$

$\Rightarrow (b + c)(a^2 + bc + ab + ac) = 0$

$\Rightarrow (b + c)(a^2 + ab + bc + ac) = 0$

$\Rightarrow (b + c)[a(a + b) + c(a + b)] = 0$

$\Rightarrow (b + c)(a + b)(c + a) = 0.$

40. Let $\dfrac{x}{(b-c)(b+c-2a)} = \dfrac{y}{(c-a)(c+a-2b)}$

$= \dfrac{z}{(a-b)(a+b-2c)} = k.$

Then, $x = k(b - c)(b + c - 2a)$

$y = k(c - a)(c + a - 2b)$

$z = k(a - b)(a + b - 2c)$

$\therefore x + y + z = k(b - c)(b + c - 2a) + k(c - a)(c + a - 2b)$
$+ k(a - b)(a + b - 2c)$

$= k(b^2 - c^2 - 2ab + 2ca) + k(c^2 - a^2 - 2bc + 2ab)$
$+ k(a^2 - b^2 - 2ca + 2bc)$

$= k(b^2 - c^2 - 2ab + 2ca + c^2 - a^2 - 2bc + 2ab + a^2 - b^2$
$- 2ca + 2bc)$

$= k \times 0 = \mathbf{0.}$

41. $a + b + c = 0$

$\Rightarrow a^2 = (b + c)^2$ or $a = -b - c$

$\therefore$ Given expression $= \dfrac{a^2}{a^2 - bc} + \dfrac{b^2}{b^2 - ca} + \dfrac{c^2}{c^2 - ab}$

$= \dfrac{(b+c)^2}{(b+c)^2 - bc} + \dfrac{b^2}{b^2 + c(b+c)} + \dfrac{c^2}{c^2 + b(b+c)}$

$= \dfrac{(b+c)^2}{b^2 + c^2 + bc} + \dfrac{b^2}{b^2 + c^2 + bc} + \dfrac{c^2}{c^2 + b^2 + bc}$

$= \dfrac{b^2 + c^2 + 2bc + b^2 + c^2}{b^2 + c^2 + bc} = \dfrac{2(b^2 + c^2 + bc)}{(b^2 + c^2 + bc)} = \mathbf{2.}$

42. $\dfrac{(x-y)^3 + (y-z)^3 + (z-x)^3}{(x^2-y^2)^3 + (y^2-z^2)^3 + (z^2-x^2)^3}$

$= \dfrac{3(x-y)(y-z)(z-x)}{3(x^2-y^2)(y^2-z^2)(z^2-x^2)}$

$\left[\begin{array}{l} \because a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc \\ \text{Here } (x - y) + (y - z) + (z - x) = 0 \\ \text{and } (x^2 - y^2) + (y^2 - z^2) + (z^2 - x^2) = 0 \end{array}\right]$

$= \dfrac{3(x-y)(y-z)(z-x)}{3(x+y)(x-y)(y+z)(y-z)(z+x)(z-x)}$

$= \dfrac{1}{(x+y)(y+z)(z+x)} = \mathbf{[(x + y)(y + z)(z + x)]^{-1}}$

43. If $a + b + c = 0$, then $a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow (a + b)(a^2 - ab + b^2) + c^3 = 3abc$

$\Rightarrow (-c)(a^2 - ab + b^2) + c^3 = 3abc$ $[\because (a + b) = -c]$

$\Rightarrow a^2 - ab + b^2 - c^2 = -3ab$

$\Rightarrow a^2 - ab + b^2 + 2ab - c^2 = -3ab + 2ab$

$= a^2 + ab + b^2 = \boldsymbol{c^2 - ab.}$

44. $\dfrac{1}{1+p+q^{-1}}+\dfrac{1}{1+q+r^{-1}}+\dfrac{1}{1+r+p^{-1}}$

$$=\frac{1}{1+p+\frac{1}{q}}+\frac{1}{1+q+\frac{1}{r}}+\frac{1}{1+r+\frac{1}{p}}$$

$$=\frac{q}{q+pq+1}+\frac{r}{r+qr+1}+\frac{p}{p+pr+1}$$

$$=\frac{q}{q+\frac{1}{r}+1}+\frac{r}{r+\frac{1}{p}+1}+\frac{p}{p+pr+1} \qquad [\because pqr=1]$$

$$=\frac{qr}{qr+1+r}+\frac{pr}{pr+1+p}+\frac{p}{p+pr+1}$$

$$=\frac{qr}{\frac{1}{p}+1+r}+\frac{pr}{pr+1+p}+\frac{p}{p+pr+1}$$

$$=\frac{pqr}{1+p+pr}+\frac{pr}{pr+1+p}+\frac{p}{p+pr+1}$$

$$=\frac{pqr+pr+p}{1+p+pr}=\frac{1+pr+p}{1+p+pr}=\mathbf{1.}$$

45. Given, $c=\dfrac{yz}{y+z}\Rightarrow cy+cz=yz\Rightarrow yz-cz=cy\Rightarrow z(y-c)=cy$

$$\Rightarrow z=\frac{cy}{y-c}$$

Also $b=\dfrac{xz}{x+z}\Rightarrow z=\dfrac{bx}{x-b}$

$$\therefore \frac{cy}{y-c}=\frac{bx}{x-b}\Rightarrow cyx-cyb=bxy-bxc$$

$$\Rightarrow cyx-cyb-bxy=-bxc$$

$$\Rightarrow -y(bx+bc-cx)=-bxc$$

$$\Rightarrow y=\frac{bxc}{bx+bc-cx}$$

Now, $a=\dfrac{xy}{x+y}\Rightarrow y=\dfrac{ax}{x-a}$.

$$\therefore \frac{bxc}{bx+bc-cx}=\frac{ax}{x-a}$$

$$\Rightarrow abx^2+abcx-acx^2=bx^2c-abcx$$

$$\Rightarrow 2abcx=x^2(bc+ac-ab)\Rightarrow x=\frac{2abc}{(bc+ac-ab)}$$

46. $a+b+c=0\Rightarrow (a+b+c)^2=0$

$$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca=0$$

$$\Rightarrow a^2+b^2+c^2=-(2ab+2bc+2ca)$$

Now, $\dfrac{a^2+b^2+c^2}{(a-b)^2+(b-c)^2+(c-a)^2}$

$$=\frac{a^2+b^2+c^2}{a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ca}$$

$$=\frac{a^2+b^2+c^2}{2(a^2+b^2+c^2)-(2ab+2bc+2ca)}$$

$$=\frac{a^2+b^2+c^2}{2(a^2+b^2+c^2)+(a^2+b^2+c^2)}$$

$$=\frac{a^2+b^2+c^2}{3(a^2+b^2+c^2)}=\mathbf{\frac{1}{3}.}$$

47. $x+y+z=2s$

Also, $(s-x)+(s-y)+(-z)=2s-(x+y+z)$
$=2s-2s=0.$

$$\Rightarrow (s-x)^3+(s-y)^3+(-z)^3-3(s-x)(s-y)(-z)=0$$

$$\left[\begin{array}{l}\because\ a+b+c=0\\ \Rightarrow a^3+b^3+c^3-3abc=0\end{array}\right]$$

$$\Rightarrow (s-x)^3+(s-y)^3+3(s-x)(s-y)(z)=z^3$$

48. Given, $x+\dfrac{1}{x}=p \qquad \Rightarrow \left(x+\dfrac{1}{x}\right)^2=p^2$

$$\Rightarrow x^2+\frac{1}{x^2}+2=p^2 \qquad \Rightarrow x^2+\frac{1}{x^2}=p^2-2$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^3=(p^2-2)^3$$

$$\Rightarrow x^6+\frac{1}{x^6}+3\left(x^2+\frac{1}{x^2}\right)=p^6-8+6p^2(p^2-2)$$

$$\Rightarrow x^6+\frac{1}{x^6}+3(p^2-2)=p^6-8+6p^2(p^2-2)$$

$$\Rightarrow x^6+\frac{1}{x^6}=\mathbf{p^6-6p^4-9p^2-2}$$

49. Given, $x+y+z=0\Rightarrow x+y=-z$

$$\Rightarrow x^2+y^2+2xy=z^2\Rightarrow x^2+y^2=z^2-2xy$$

$$\therefore \frac{1}{x^2+y^2-z^2}=\frac{1}{z^2-2xy-z^2}=\frac{1}{-2xy}=-\frac{1}{2xy}$$

Similarly, $\dfrac{1}{y^2+z^2-x^2}=-\dfrac{1}{2yz}$ and $\dfrac{1}{z^2+x^2-y^2}=-\dfrac{1}{2zx}$

$$\therefore \frac{1}{x^2+y^2-z^2}+\frac{1}{y^2+z^2-x^2}+\frac{1}{z^2+x^2-y^2}$$

$$=-\frac{1}{2xy}-\frac{1}{2yz}-\frac{1}{2zx}=-\frac{1}{2}\left[\frac{z+x+y}{xyz}\right]=\mathbf{0}$$

$[\because x+y+z=0]$

50. $x+y+z=0\Rightarrow x=-y-z$ ...(*i*)
$y=-x-z$ ...(*ii*)
$z=-x-y$ ...(*iii*)

Now, $\dfrac{x^2}{2x^2+yz}+\dfrac{y^2}{2y^2+zx}+\dfrac{z^2}{2z^2+xy}$

$$=\frac{x^2}{x^2+x.x+yz}+\frac{y^2}{y^2+y.y+zx}+\frac{z^2}{z^2+z.z+xy}$$

$$= \frac{x^2}{x^2 + x(-y-z) + yz} + \frac{y^2}{y^2 + y(-x-z) + zx} + \frac{z^2}{z^2 + z(-x-y) + xy}$$

$$= \frac{x^2}{x^2 - xy - xz + yz} + \frac{y^2}{y^2 - yx - zy + zx} + \frac{z^2}{z^2 - zx - zy + xy}$$

$$= \frac{x^2}{x(x-y) - z(x-y)} + \frac{y^2}{y(y-x) - z(y-x)} + \frac{z^2}{z(z-x) - y(z-x)}$$

$$= \frac{x^2}{(x-y)(x-z)} + \frac{y^2}{(y-x)(y-z)} + \frac{z^2}{(z-x)(z-y)}$$

$$= -\frac{x^2}{(x-y)(z-x)} - \frac{y^2}{(x-y)(y-z)} - \frac{z^2}{(z-x)(y-z)}$$

$$= -\left[\frac{x^2(y-z) + y^2(z-x) + z^2(x-y)}{(x-y)(y-z)(z-x)}\right]$$

$$= -\left[\frac{-(x-y)(y-z)(z-x)}{(x-y)(y-z)(z-x)}\right] = -(-1) = \mathbf{1}.$$

Factorising the numerator.

51. Given, $(b + c - a)x = (c + a - b)y = (a + b - c)z = 2$

$$\Rightarrow x = \frac{2}{(b+c-a)};\ y = \frac{2}{(c+a-b)};\ z = \frac{2}{(a+b-c)}$$

$$\therefore \frac{1}{x} = \frac{b+c-a}{2};\ \frac{1}{y} = \frac{c+a-b}{2};\ \frac{1}{z} = \frac{a+b-c}{2}$$

$$\therefore \left(\frac{1}{y} + \frac{1}{z}\right) = \left(\frac{c+a-b}{2} + \frac{a+b-c}{2}\right) = \frac{2a}{2} = a$$

$$\left(\frac{1}{z} + \frac{1}{x}\right) = \left(\frac{a+b-c}{2} + \frac{b+c-a}{2}\right) = \frac{2b}{2} = b$$

$$\left(\frac{1}{x} + \frac{1}{y}\right) = \left(\frac{b+c-a}{2} + \frac{a+c-b}{2}\right) = \frac{2c}{2} = c$$

$$\therefore \left(\frac{1}{y} + \frac{1}{z}\right)\left(\frac{1}{z} + \frac{1}{x}\right)\left(\frac{1}{x} + \frac{1}{y}\right) = a\,.\,b\,.\,c = \boldsymbol{abc}.$$

52. $a^x \,.\, a^y \,.\, a^z = (x + y + z)^{x+y+z}$

$\Rightarrow a^{x+y+z} = (x + y + z)^{x+y+z}$

$\Rightarrow a = (x + y + z)$

Now, $(x + y + z)^y = a^x$ (given)

$\Rightarrow (x + y + z)^y = (x + y + z)^x \Rightarrow y = x$

Similarly, $y = z$ and $z = x$.

$$\therefore x = y = z = \frac{x+y+z}{3} = \frac{\boldsymbol{a}}{\mathbf{3}}.$$

53. Given, $x + \frac{1}{x} = a.$

Now, $x^3 + x^2 + \frac{1}{x^3} + \frac{1}{x^2} = \left(x^3 + \frac{1}{x^3}\right) + \left(x^2 + \frac{1}{x^2}\right)$

$$= \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) + \left(x + \frac{1}{x}\right)^2 - 2$$

$= a^3 - 3a + a^2 - 2 = \boldsymbol{a^3 + a^2 - 3a - 2}.$

54. If $x^{1/3} + y^{1/3} + z^{1/3} = 0$, then

$(x^{1/3})^3 + (y^{1/3})^3 + (z^{1/3})^3 = 3x^{1/3}y^{1/3}z^{1/3}$

$$\left[\because a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc\right]$$

$\Rightarrow x + y + z = 3x^{1/3}y^{1/3}z^{1/3}$

Now taking the cube of both the sides, we have

$(x + y + z)^3 = (3x^{1/3}y^{1/3}z^{1/3})^3 = \mathbf{27}\boldsymbol{xyz}.$

55. Given expression

$$= \frac{2a(b+c)(c+a) + 2b(a+b)(c+a) + 2c(a+b)(b+c)}{(a+b)(b+c)(c+a)} + \frac{(b-c)(c-a)(a-b)}{(b+c)(c+a)(a+b)}$$

$$= \frac{2a(bc + c^2 + ab + ac) + 2b(ac + bc + a^2 + ab) + 2c(ab + b^2 + ac + bc) + (bc - c^2 - ab + ac)(a-b)}{(b+c)(c+a)(a+b)}$$

$$= \frac{2abc + 2ac^2 + 2a^2b + 2a^2c + 2abc + 2b^2c + 2ba^2 + 2ab^2 + 2abc + 2cb^2 + 2ac^2 + 2bc^2 + abc - ac^2 - a^2b + a^2c - b^2c + bc^2 + ab^2 - abc}{(b+c)(c+a)(a+b)}$$

$$= \frac{6abc + 3ac^2 + 3a^2b + 3a^2c + 3b^2c + 3ab^2 + 3bc^2}{(b+c)(c+a)(a+b)}$$

$$= \frac{3[2abc + ac^2 + a^2b + a^2c + b^2c + ab^2 + bc^2]}{(b+c)(c+a)(a+b)}$$

$$= \frac{3(b+c)(c+a)(a+b)}{(b+c)(c+a)(a+b)} = \mathbf{3}.$$

SELF ASSESSMENT SHEET

1. $(x^n - a^n)$ is divisible by $(x - a)$

(*a*) for all values of n (*b*) for even values of n

(*c*) for odd values of n (*d*) only for prime values of n

2. If $(x + 1)$ is a factor of $x^4 + 9x^3 + 7x^2 + 9ax + 5a^2$, then :

(*a*) $a = 137$ (*b*) $5a^2 - 9a - 1 = 0$

(*c*) $5a^2 + 9a + 17 = 0$ (*d*) $a = \sqrt{131}$

(CDS 2004)

3. When $x^3 + 2x^2 + 4x + b$ is divided by $(x + 1)$, the quotient is $x^2 + ax + 3$ and the remainder is $-3 + 2b$. What are the values of a and b respectively ?

(*a*) 1, 0 (*b*) –1, 0 (*c*) 1, 1 (*d*) –1, –1

(CDS 2005)

4. If $x^3 + px + q$ and $x^3 + qx + p$ have a common factor, then which of the following is correct ?

(*a*) $p + q = 0$ (*b*) $p + q - 1 = 0$

(*c*) $(p + q + 1) = 0$ (*d*) $p - q + 1 = 0$

(*CDS 2005*)

5. $(2x - 3y)^3 + (3y - 4z)^3 + (4z - 2x)^3$ can be factorised into which one of the following ?

(*a*) $(2x + 3y + 4z)(2x - 3y - 4z)$

(*b*) $(2x + 3y - 4z)(2x - 3y + 4z)$

(*c*) $(2x - 3y)(3y - 4z)(4z - 2x)$

(*d*) $6(2x - 3y)(3y - 4z)(2z - x)$ **(*CDS 2005*)**

6. If $ax^3 + bx^2 + x - 6$ has $(x + 2)$ as a factor and leaves a remainder 4, when divided by $(x - 2)$, the value of a and b respectively are :

(*a*) 1, –2 (*b*) 2, 1 (*c*) 0, 2 (*d*) 1, –1

7. If $4x^2 - 6x + m$ is divisible by $x - 3$, which one of the following is the greatest divisor of m ?

(*a*) 9 (*b*) 12 (*c*) 18 (*d*) 36

(*CDS 2006*)

8. If $y = x + \frac{1}{x}$, then $x^4 + x^3 - 4x^2 + x + 1 = 0$ can be reduced to which one of the following ? $(x \neq 0)$

(*a*) $y^2 + y - 2 = 0$ (*b*) $y^2 + y - 4 = 0$

(*c*) $y^2 + y - 6 = 0$ (*d*) $y^2 + y + 6 = 0$

(*CDS 2006*)

9. If $x^2 = y + z$, $y^2 = z + x$, $z^2 = x + y$, then what is the value of:

$$\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} ?$$

(*a*) 1 (*b*) 0 (*c*) –1 (*d*) 2

(*CDS 2007*)

10. $\dfrac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$ on simplification is equal to :

(*a*) 1 (*b*) $(a - b)(b - c)(c - a)$

(*c*) $(a + b)(b + c)(c + a)$ (*d*) 0

ANSWERS

1. (*a*)	**2.** (*b*)	**3.** (*a*)	**4.** (*c*)	**5.** (*d*)	**6.** (*c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*a*)	**10.** (*c*)

HINTS AND SOLUTIONS

2. Let $f(x) = x^4 + 9x^3 + 7x^2 + 9ax + 5a^2$.

If $(x + 1)$ is a factor of $f(x)$, then

$f(-1) = 0 \Rightarrow (-1)^4 + 9(-1)^3 + 7(-1)^2 + 9a(-1) + 5a^2 = 0$

$\Rightarrow 1 - 9 + 7 - 9a + 5a^2 = 0 \Rightarrow \mathbf{5a^2 - 9a - 1 = 0.}$

3. Let $f(x) = x^3 + 2x^2 + 4x + b$.

When divided by $(x + 1)$, the remainder $= f(-1)$

Given, remainder $= -3 + 2b$

$\therefore -3 + 2b = f(-1) = (-1)^3 + 2(-1)^2 + 4(-1) + b$

$\Rightarrow -3 + 2b = -1 + 2 - 4 + b$

$\Rightarrow -3 + 2b = -3 + b$.

This is only possible when $\mathbf{b = 0.}$

$\therefore f(x) = x^3 + 2x^2 + 4x$.

Now dividing $f(x)$ by $(x + 1)$, we see that

```
         x² + x + 3
x + 1 ) x³ + 2x² + 4x
        x³ + x²
       –    –
       ----------
             x² + 4x
             x² + x
            –    –
            ---------
                  3x
                  3x + 3
                 –    –
                 --------
                      –3
                 --------
```

$\therefore$ Quotient $= x^2 + ax + 3 = x^2 + x + 3 \Rightarrow a = \mathbf{1}$.

4. Let the common factor be $x - \alpha$, then $x = \alpha$ will make the given expressions zero, *i.e.*,

$$\alpha^2 + p\alpha + q = 0$$
$$\alpha^2 + q\alpha + p = 0$$

Solving by the rule of cross-multiplication, we have

$$\frac{\alpha^2}{p^2 - q^2} = \frac{\alpha}{q - p} = \frac{1}{q - p}$$

From, last two relation, $\alpha = 1$.

$$\Rightarrow \alpha^2 = \frac{p^2 - q^2}{q - p} \Rightarrow 1 = -(p + q) \Rightarrow \mathbf{p + q + 1 = 0.}$$

5. Since $(2x - 3y) + (3y - 4z) + (4z - 2x) = 0$, therefore,

$(2x - 3y)^3 + (3y - 4z)^3 + (4z - 2x)^3$

Take out 2 common

$= 3.(2x - 3y)(3y - 4z)(4z - 2x)$

$= 6.(2x - 3y)(3y - 4z)(2z - x)$.

6. Let $f(x) = ax^3 + bx^2 + x - 6$

$(x + 2)$ is a factor of $f(x) \Rightarrow f(-2) = 0$

$\therefore f(-2) = -8a + 4b - 2 - 6 = 0$

$\Rightarrow -8a + 4b - 8 = 0 \Rightarrow -2a + b = 2$ ...(*i*)

$(x - 2)$ leaves a remainder 4, when dividing $f(x) \Rightarrow f(2) = 4$

$\therefore f(2) = 8a + 4b + 2 - 6 = 4 \Rightarrow 8a + 4b - 8 = 0$

$\Rightarrow 2a + b = 2$ ...(*ii*)

$\therefore$ From (*i*) and (*ii*) $b = \mathbf{2}$, $a = \mathbf{0}$.

7. If $f(x) = 4x^2 - 6x + m$ is divisible by $(x - 3)$, then $f(3) = 0$

$\Rightarrow 4(3)^2 - 6.3 + m = 0 \Rightarrow 36 - 18 + m = 0 \Rightarrow m = -18$.

The greatest divisor of m = **18**.

8. $x^4 + x^3 - 4x^2 + x + 1 = 0$

$$\Rightarrow x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$$

$$\Rightarrow x^2 + \frac{1}{x^2} + x + \frac{1}{x} - 4 = 0$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 - 2 + \left(x + \frac{1}{x}\right) - 4 = 0$$

$$\Rightarrow y^2 + y - 6 = 0 \qquad \left(\because x + \frac{1}{x} = y\right)$$

9. $x^2 = y + z \Rightarrow x^2 + x = x + y + z$

$$\Rightarrow x(x + 1) = x + y + z \Rightarrow \frac{x}{x + y + z} = \frac{1}{x + 1}$$

Similarly, $\frac{1}{y + 1} = \frac{y}{x + y + z}$ and $\frac{1}{z + 1} = \frac{z}{x + y + z}$

$$\therefore \frac{1}{x + 1} + \frac{1}{y + 1} + \frac{1}{z + 1} = \frac{x}{x + y + z} + \frac{y}{x + y + z}$$

$$+ \frac{z}{x + y + z} = \frac{x + y + z}{x + y + z} = \mathbf{1.}$$

10. Use the identity, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

WHY MODULUS JUNIOR COLLEGE ?

Best IITJEE Faculty : Team led by an Alumni of IIT Delhi,**MODULUS** has expert faculty to provide IITJEE coaching, mentoring and counselling to the students to reach their goal.

Exceptional Teaching Methodology : Concept Oriented Teaching rather than Rote Learning. Sufficient self-study time is given to grasp the concepts and to apply them to solve the difficult problems. Classes will be interactive to keep students curious and interested in subject.

"I listen - I forget,
I read -I remember,
I do - I understand"

following this saying, measures like asking students to teach some subtopic will be taken.

Scientifically Researched Program : IITJEE experts /toppers recommend for every one hour of teaching, one hour of self study should be given. Triumph program is designed keeping this in mind.

Optimum Batch Size : Batch size of 30-35 students to provide individual attention to each and every student.

Excellent Study material : Complete, well structured and comprehensive study material prepared by our Subject Matter Experts will be provided.

Regular Exams : IITJEE(JEE MAINS/ADVANCE), BITSAT & EAMCET pattern exams will be conducted weekly to test students followed by test analysis to diagnose the students weak areas.

Students Mentorship Program : Students will be assigned Mentors to resolve their day to day problems. MENTORS will follow up with students on regular basis to monitor their progress and will help them to stay focused on their goal.

Overall Personality Development : Regular sessions of YOGA, MEDITATION, SPORTS, Moral and Ethical Behavior will be conducted for students holistic development.

Regular PTM & Motivational seminars: Regular Parent - Teacher meetings will be conducted to discuss the performance of their ward. Regular motivational seminars by IIT Alumni , IIT JEE Experts to instigate the deep desire in students to work hard to get admissions into world class institutes.

Career Guidance to students after completion of their Intermediate course.

Transport Facility: Transport Facility will be provided to needy students.

Hostel Facility: Separate hostel facility for BOYS and GIRLS will be provided.

MODULUS Junior College

WEEK DAYS / WEEKENDS
Class Room Programs

7th
For 7th Class Students
ENRICH TWO YEAR

8th
For 8th Class Students
ENRICH ONE YEAR

The main aim of the programme is to enrich logical thinking, IQ besides laying strong foundation in MATHS, PHYSICS, CHEMISTRY.
This program also helps students to excel in school exams.

9th
For 9th Class Students
REFINE TWO YEAR

10th
For 10th Class Students
REFINE ONE YEAR

The goal of this program is to lay strong foundation for the most prestigious competitive exams like NTSE, IIT-JEE, KVPY, NSEJS and other Olympiads.
This also helps students to perform exceptionally in board exams.

11th
For 11th Class Students
TRIUMPH TWO YEAR

12th
For 12th Class Students
TRIUMPH ONE YEAR

The above programs will help the student to perform well in IIT-JEE Mains and Advanced exams. This also equip the student excel in other engineering and board exams.

ADDRESS
OPP. ZENAS INTERNATIONAL SCHOOL , Near Miyapur Metro First Pillar ,
Mathrusri Nagar , Miyapur ,Hyderabad.

www.modulushyd.com www.facebook.com/modulushyd modulushyd@gmail.com