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Previous Years JEE ADVANCED Rank VS % of Marks

RANK	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	Avg %	2021 (M. Marks 360) Marks	2021 (M. Marks 360) Rank	2021 (M. Marks 360) Marks	2021 (M. Marks 360) Rank
1	85.48	91.67	96.01	92.22	92.78	93.06	86.02	92.62	93.61	93.01	88.89	91.40	348	1	196	1544
10	81.39	85.42	88.48	85.83	86.94	83.92	78.49	86.88	86.11	82.80	81.06	84.99	324	8	185	2043
100	70.76	79.79	78.68	76.39	77.50	69.84	68.61	83.33	74.72	72.85	71.72	74.93	323	13	178	2395
500	62.17	72.08	69.36	66.67	68.61	60.32	54.57	77.32	64.44	61.02	59.34	65.08	313	25	160	3542
1000	57.87	68.13	64.46	61.67	63.33	56.75	49.46	72.95	58.59	55.65	53.28	60.22	310	27	154	4137
2000	52.97	62.50	58.33	55.56	57.22	49.21	43.55	67.48	53.06	50.00	46.46	54.21	302	46	145	5005
3000	49.69	59.17	54.41	51.94	53.61	45.44	42.74	63.38	49.17	46.77	42.42	50.79	297	66	135	6180
4000	47.03	56.67	51.72	49.17	51.11	42.66	37.90	60.38	46.39	44.09	39.39	47.86	296	70	131	6750
5000	44.99	54.38	49.51	47.22	48.89	40.48	36.02	57.65	43.89	41.94	37.12	45.64	286	94	126	7516
6000	43.35	52.17	47.55	45.56	47.22	38.69	34.41	55.45	41.94	40.32	35.10	43.85	266	201	121	8494
7000	41.92	51.04	45.83	43.89	45.56	37.10	33.33	53.55	40.28	38.71	33.59	42.25	254	299	109	10848
8000	40.49	49.79	43.38	42.50	44.44	35.91	31.99	51.63	38.61	37.63	32.32	40.79	245	395	102	12874
9000	39.47	48.54	43.14	41.11	43.06	34.52	30.91	50.27	37.22	36.29	30.81	39.58	239	471	100	13215
10000	38.85	47.71	41.91	40.00	41.94	33.33	29.84	48.90	36.11	35.22	29.80	38.51	231	600	85	18547
QUAL%	38.85	47.71	34.55	33.88	35.00	23.81	20.16	22.50	25.00	25.00	17.42	29.44	212	1006	79	21157

3 Quadratic Equations

KEY FACTS

1. An equation in which the highest power of the variable is 2 is called a quadratic equation.
$ax^2 + bx + c = 0$, where a, b, c are constants is a general quadratic equation and $a \neq 0$, and $a, b, c \in R$.

2. Solving a Quadratic Equation : To find the roots of a quadratic equation is called solving a quadratic equation.

(*a*) **Method I :** ***Factorising the quadratic equation into linear factors.***

The quadratic expression $ax^2 + bx + c = 0$ can be expressed as a product of two linear factors as the degree of the algebraic expression here is **2.**

Let $ax^2 + bx + c = (mx + n)(ex + f)$, where $m \neq 0$, $e \neq 0$.

Then, $ax^2 + bx + c = 0 \Rightarrow (mx + n)(ex + f) = 0$

$\Rightarrow (mx + n) = 0$ or $(ex + f) = 0$

$\Rightarrow x = -\dfrac{n}{m}$ or $x = -\dfrac{f}{e}$

$\therefore$ The two roots of $ax^2 + bx + c = 0$ are $\dfrac{-n}{m}$ and $\dfrac{-f}{e}$.

(*b*) **Method II :** ***Using the formula.***

$ax^2 + bx + c = 0$ $\quad (a \neq 0)$

$\Rightarrow ax^2 + bx = -c$ $\quad$ (Transposing the constant term)

$\Rightarrow x^2 + \dfrac{b}{a}x = -\dfrac{c}{a}$ $\quad$ (Dividing by the coefficient of x^2)

$\Rightarrow x^2 + \dfrac{b}{a}x + \dfrac{b^2}{4a^2} = \dfrac{b^2}{4a^2} - \dfrac{c}{a}$ $\quad$ $\left(\text{Adding} \left(\dfrac{b}{2a}\right)^2 \text{ on both the sides to make LHS a perfect square}\right)$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \Rightarrow x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The formula

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Hence, the roots of the equation $ax^2 + bx + c = 0$ are $\boldsymbol{\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}}$ and $\boldsymbol{\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}}$

These two values are called the roots of the equation and are also called the **zeros of the function defined** by $f(x) = ax^2 + bx + c$.

3. Equations Reducible to Quadratic Equations

Type I : $ax^{2n} + bx^n + c = 0$

In this type of an equation, we put $x^n = y$. So, $ax^{2n} + bx^n + c = 0$ reduces to $ay^2 + by + c = 0$.

Now solve for y and hence for x.

Example. ***Solve : $4^{1+x} + 4^{1-x} = 10$ for x.***

Sol. $4^{1+x} + 4^{1-x} = 10 \Rightarrow 4^1.4^x + 4^1.4^{-x} = 10 \Rightarrow 4.4^x + \frac{4}{4^x} = 10$

$\Rightarrow 4.4^{2x} + 4 = 10 \times 4^x \Rightarrow 4.4^{2x} - 10.4^x + 4 = 0.$

Let $4^x = y$. Then, the given equation becomes $4y^2 - 10y + 4 = 0$

$\Rightarrow 2y^2 - 5y + 2 = 0 \Rightarrow (y-2)(2y-1) = 0 \Rightarrow y = 2$ or $\frac{1}{2}$.

$\Rightarrow 4^x = 2$ or, $4^x = +\frac{1}{2} \Rightarrow 2^{2x} = 2^1$ or $2^{2x} = 2^{-1} \Rightarrow 2x = 1$ or $2x = -1 \Rightarrow x = \mathbf{\frac{1}{2}}$ **or** $\mathbf{\frac{-1}{2}}$.

Type II : $az + \frac{b}{z} = c$, *where a, b, c are constants.*

Example. ***If*** $\sqrt{\frac{2x^2+x+2}{x^2+3x+1}} + 2\sqrt{\frac{x^2+3x+1}{2x^2+x+2}} - 3 = 0$, ***find x.***

Sol. Let $\sqrt{\frac{2x^2+x+2}{x^2+3x+1}} = y$. Then, $y + 2 \times \frac{1}{y} - 3 = 0$

$\Rightarrow y^2 - 3y + 2 = 0 \Rightarrow (y-1)(y-2) = 0 \Rightarrow y = 1$ or 2

$\therefore y = 1 \Rightarrow \sqrt{\frac{2x^2+x+2}{x^2+3x+1}} = 1 \Rightarrow \frac{2x^2+x+2}{x^2+3x+1} = 1 \Rightarrow 2x^2 + x + 2 = x^2 + 3x + 1$

$\Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1.$

$y = 2 \Rightarrow \sqrt{\frac{2x^2+x+2}{x^2+3x+1}} = 2 \Rightarrow \frac{2x^2+x+2}{x^2+3x+1} = 4$

$\Rightarrow 2x^2 + x + 2 = 4x^2 + 12x + 4 \Rightarrow 2x^2 + 11x + 2 = 0$

$\therefore x = \frac{-11 \pm \sqrt{121 - 4\times2\times2}}{2\times2} = \frac{-11\pm\sqrt{121-16}}{4} = \frac{-11\pm\sqrt{105}}{4}$. $\therefore$ $\mathbf{x = 1, \frac{-11\pm\sqrt{105}}{4}}$.

Type III : *Equations of type (x + a) (x + b) (x + c) (x + d) + k = 0, where the sum of two of the quantities a, b, c, d is equal to the sum of the other two.*

Example. ***(x + 1) (x + 2) (x + 3) (x + 4) + 1 = 0***

Sol. $[(x+1)(x+4)][(x+2)(x+3)] + 1 = 0$ $(\because 1 + 4 = 2 + 3 = 5)$

$\Rightarrow (x^2 + 5x + 4)(x^2 + 5x + 6) + 1 = 0$

Let $x^2 + 5x = y$. Then, $(y+4)(y+6) + 1 = 0$

$\Rightarrow y^2 + 10y + 24 + 1 = 0 \Rightarrow y^2 + 10y + 25 = 0 \Rightarrow (y+5)^2 = 0 \Rightarrow y = -5$

$\therefore x^2 + 5x = -5 \Rightarrow x^2 + 5x + 5 = 0. \Rightarrow x = \frac{-5\pm\sqrt{25-20}}{2} = \mathbf{\frac{-5\pm\sqrt{5}}{2}}$.

4. Important Properties of Inequalities

1. *An inequality will still hold after each side has been increased, diminished, multiplied or divided by the same **positive** quantity.*
2. *In an inequality any term may be transposed from one side to the other if its sign is changed.*
3. *Both the sides of an inequality can be multiplied or divided by the same **negative** number **by reversing the sign of inequality.***

5. Nature of Roots. A quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, has two roots which by the quadratic formula are as under :

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$

The expression $\boldsymbol{b^2 - 4ac}$ is called the **discriminant.**

Examining the nature of the roots means to see what type of roots the equation has, that is, whether they are **real or imaginary, real or irrational, equal or unequal**. The nature of the roots depends entirely on the value of the discriminant $\boldsymbol{D = b^2 - 4ac}$

Thus, if a, b, c are rational, then

I. If $\boldsymbol{D = b^2 - 4ac > 0}$ (*i.e*, *positive*), the roots are **real** and **unequal.**

Also,

(a) If $\boldsymbol{D = b^2 - 4ac}$ is a **perfect square,** the roots are **rational.**

(b) If $\boldsymbol{D = b^2 - 4ac}$ is not a **perfect square,** the roots are **irrational.**

(c) If $\boldsymbol{D = b^2 - 4ac = 0}$**,** the roots are **equal,** each being equal to $\boldsymbol{\frac{-b}{2a}}$.

So, $\boldsymbol{ax^2 + bx + c = 0}$ **is a perfect square if** $\boldsymbol{D = 0}$**.**

II. If $\boldsymbol{D = b^2 - 4ac < 0}$ **(***i.e.*, **negative),** the roots are **imaginary** (*complex*).

Example. ***Examine the nature of the roots of the equations:***

(i) $\boldsymbol{2x^2 + 2x + 3 = 0}$ ***(ii)*** $\boldsymbol{2x^2 - 7x + 3 = 0}$

(iii) $\boldsymbol{x^2 - 5x - 2 = 0}$ ***(iv)*** $\boldsymbol{4x^2 - 4x + 1 = 0.}$

Sol. (i) $2x^2 + 2x + 3 = 0$ (Here, $a = 2$, $b = 2$, $c = 3$)

$\therefore$ $D = b^2 - 4ac = (2)^2 - 4 \times 2 \times 3 = 4 - 24 = \boldsymbol{-20 < 0}$

Hence, roots are **imaginary.**

(ii) $2x^2 - 7x + 3 = 0$ (Here, $a = 2$, $b = -7$, $c = 3$)

$\therefore$ $D = b^2 - 4ac = 49 - 24 = \boldsymbol{25 > 0}$ and **a perfect square**

Hence, roots are **real and rational.**

(iii) $x^2 - 5x - 2 = 0.$ (Here, $a = 1$, $b = -5$, $c = -2$)

$\therefore$ $D = b^2 - 4ac = 25 + 8 = \boldsymbol{33 > 0}$ and **not a perfect square**

Hence, roots are **real and irrational.**

(iv) $4x^2 - 4x + 1 = 0$ (Here, $a = 4$, $b = -4$, $c = 1$)

$\therefore$ $D = b^2 - 4ac = 16 - 16 = \boldsymbol{0.}$

Hence, roots are **real and equal.**

6. Sum and Product of Roots:

If the two roots of the quadratic equation $ax^2 + bx + c = 0$ obtained by the quadratic formula be denoted by α and β, then we have

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}, \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\therefore \text{ Sum of roots} = \alpha + \beta = -\frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-2b}{2a} = \boldsymbol{\frac{-b}{a}}$$

$$\text{Product of roots} = \alpha\beta = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right) \times \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{(-b)^2 - (\sqrt{b^2-4ac})^2}{4a^2} = \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{4ac}{4a^2} = \boldsymbol{\frac{c}{a}}.$$

Thus, **Sum of roots** $= \dfrac{-\textbf{Coeff. of } x}{\textbf{Coeff. of } x^2}$; **Product of roots** $= \dfrac{\textbf{Constant term}}{\textbf{Coeff. of } x^2}$

Thus, if α, β be the roots of the equation $6x^2 - 5x + 7 = 0$, then $\alpha + \beta = -(-5/6) = 5/6$, $\alpha\beta = 7/6$.

7. Values of the Symmetric Functions of the Roots

If α, β be the roots of a given quadratic equation and we wish to find the value of a symmetric function of α and β, **we can do so by proceeding as follows :**

Method:

Step I. *Write the values of* $\alpha + \beta$ *and* $\alpha\beta$ *from the given equation.*

Step II. *Express the given function in terms of* $\alpha + \beta$ *and* $\alpha\beta$.

Step III. *Substitute the values of* $\alpha + \beta$ *and* $\alpha\beta$ *from* ***step 1.***

Caution:Do not find the values of α and β separately.

The following algebraic relations can be very useful :

1. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
2. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
3. $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = (\alpha + \beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$
4. $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta] = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
5. $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = (\alpha - \beta)[(\alpha - \beta)^2 + 3\alpha\beta] = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$
6. $\alpha^4 - \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$
7. $\alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2) = (\alpha - \beta)(\alpha + \beta)(\alpha^2 + \beta^2) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\,(\alpha+\beta)[(\alpha+\beta)^2 - 2\alpha\beta]$

8. Formation of Equations with given Roots :

Suppose we have to form the equation whose roots are α and β. Then, as $x = \alpha$, $x = \beta$ are the roots of the equation, so $(x - \alpha) = 0$ and $(x - \beta) = 0$

$\therefore (x - \alpha)(x - \beta) = 0$

$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow$ **x^2 – (Sum of roots) x + Product of roots = 0.**

Thus, the equation whose roots are 5 and 7 is $x^2 - (5 + 7)x + 5 \times 7 = 0 \quad \Rightarrow x^2 - 12x + 35 = 0$.

9. To find the condition when a relation between the two roots is given

Step I. *Let one root be* α. *Write the other root using the given relation.*

Step II. *Write the sum and product of the roots.*

Step III. *Eliminate* $\boldsymbol{\alpha}$ *from the two relations obtained in Step II.*

Ex. ***Find the condition that one root of $ax^2 + bx + c = 0$ may be four times the other.***

Sol. Let the roots be α and 4α. Then,

$$\alpha + 4\alpha = 5\alpha = -\frac{b}{a} \qquad \ldots(i)$$

$$\alpha \,.\, 4\alpha = 4\alpha^2 = \frac{c}{a} \qquad \ldots(ii)$$

From (i) $\alpha = -\dfrac{b}{5a}$

$\therefore$ From (ii) $4.\left(-\dfrac{b}{5a}\right)^2 = \dfrac{c}{a} \Rightarrow \dfrac{4b^2}{25a^2} = \dfrac{c}{a} \Rightarrow$ $\mathbf{4b^2 = 25\,ac.}$

10. Special Roots:
For a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, $a, b, c \in R$ whose roots are α and β.

(*a*) **Reciprocal roots**

If $\alpha = \frac{1}{\beta}$, then $\alpha\beta = 1 \Rightarrow \alpha\beta = \frac{c}{a} = 1 \Rightarrow$ $\boldsymbol{c = a}$

Thus the roots of a quadratic equation will be reciprocal of each other if **coefficient of x^2 = constant term.**

(*b*) **Zero roots**

Case I : When one root is zero, say $\alpha = 0$.

Then, $\alpha\beta = 0 \Rightarrow \frac{c}{a} = 0, \Rightarrow$ $\boldsymbol{c = 0}$ as $a \neq 0$

Case II : When both roots are zero, *i.e*, $\alpha = 0, \beta = 0$

Then, $\alpha + \beta = 0$ and $\alpha\beta = 0$

$\Rightarrow \quad -\frac{b}{a} = 0$ and $\frac{c}{a} = 0 \quad \Rightarrow$ $\boldsymbol{b = 0}$ **and** $\boldsymbol{c = 0}$ as $a \neq 0$.

(*c*) **Infinite roots**

Let α, β be the roots of $ax^2 + bx + c = 0$ ...(*i*)

Then, the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$ is $cx^2 + bx + a = 0$ ...(*ii*)

Now if one root of (*ii*) is zero then $a = 0 \Rightarrow$ the corresponding root of (*i*) is $\frac{1}{0} = \infty$.

If both the roots of (*ii*) are zero, then $a = 0$, $b = 0 \Rightarrow$ both the corresponding roots of (*i*) are infinitely large.

Thus, For **one root** to be **infinite,** $\boldsymbol{a = 0}$ **;**

For **both roots** to be **infinite,** $\boldsymbol{a = 0, b = 0}$

11. Signs of the Roots

(***a***) **Positive roots : Both the roots will be positive if $\boldsymbol{\alpha + \beta}$ and $\boldsymbol{\alpha\beta}$ are both positive, *i.e*, $\boldsymbol{-\frac{b}{a}}$ and $\boldsymbol{\frac{c}{a}}$ are positive. It will be so when *b* and *a* are of opposite signs and *c* and *a* are of the same sign.**

(***b***) **Negative roots :** Both the roots will be negative if $\alpha + \beta$ is negative and $\alpha\beta$ is positive, *i.e.*, $-\frac{b}{a}$ is negative and $\frac{c}{a}$ is positive, *i.e.*, when ***a*, *b* and *c* all have the same sign.**

(***c***) **Roots of opposite signs.** It will occur when $\alpha\beta$ is negative, *i.e.*, ***c*** **and** ***a*** **are of opposite signs.**

(***d***) **Roots equal in magnitude but opposite in sign.** It will occur if $\alpha + \beta = 0$, *i.e.*, $-\frac{b}{a} = 0$, *i.e.* $b = 0$.

For real solutions the signs of c and a should be opposite.

12. Common Roots

1. **To find the condition that two quadratic equations may have one common root.**

Let the two quadratic equations be $ax^2 + bx + c = 0$, $a_1 x^2 + b_1 x + c_1 = 0$ and let α be their common root. Then,

$$a\alpha^2 + b\alpha + c = 0 \qquad ...(i)$$

$$a_1\alpha^2 + b_1\alpha + c_1 = 0 \qquad ...(ii)$$

Solving them by the rule of cross-multiplication, we have, $\frac{\alpha^2}{bc_1 - cb_1} = \frac{\alpha}{ca_1 - ac_1} = \frac{1}{ab_1 - a_1 b}$

$$\alpha^2 = \frac{bc_1 - cb_1}{ab_1 - a_1 b}, \alpha = \frac{ca_1 - ca_1}{ab_1 - a_1 b}$$

$$\Rightarrow \qquad \frac{bc_1 - cb_1}{ab_1 - a_1 b} = \frac{(ca_1 - ac_1)^2}{(ab_1 - a_1 b)^2} \Rightarrow (bc_1 - cb_1)(ab_1 - a_1 b) = (ca_1 - ac_1)^2$$

2. To find the condition that the two quadratic equations may have both the roots common.

Let the common roots of the equations $ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$ be α and β. Then,

From first equation, $\alpha + \beta = -\dfrac{b}{a}, \alpha\beta = \dfrac{c}{a}$

From second equation, $\alpha + \beta = -\dfrac{b_1}{a_1}, \alpha\beta = \dfrac{c_1}{a_1}$

So, $-\dfrac{b}{a} = -\dfrac{b_1}{a_1}$ and $\dfrac{c}{a} = +\dfrac{c_1}{a_1} \Rightarrow \boldsymbol{\dfrac{a}{a_1} = \dfrac{b}{b_1} = \dfrac{c}{c_1}}$.

13. Transformed Equations

If $f(x) = ax^2 + bx + c = 0$ is a quadratic equation, then the equation whose roots are the :

(*a*) **Reciprocals of the roots of $f(x) = 0$ is $f\left(\dfrac{1}{x}\right) = 0$** *i.e.,* $\dfrac{a}{x^2} + \dfrac{b}{x} + c = 0 \Rightarrow \boldsymbol{cx^2 + bx + a = 0.}$

(*b*) **Roots of $f(x) = 0$, each increased by a constant k is $f(x - k) = 0$,** *i.e.,* $\boldsymbol{a(x - k)^2 + b(x - k) + c = 0.}$

(*c*) **Roots of $f(x) = 0$, each decreased by a constant k is $f(x + k) = 0$,** *i.e.,* $\boldsymbol{a(x + k)^2 + b(x + k) + c = 0.}$

(*d*) **Roots of $f(x) = 0$ with signs changed** is $f(-x) = 0$, *i.e.* $a(-x)^2 + b(-x) + c = 0 \Rightarrow \boldsymbol{ax^2 - bx + c = 0.}$

(*e*) **Roots of $f(x) = 0$ with each multiplied by $k \neq 0$ is $f\left(\dfrac{x}{k}\right) = 0$** *i.e.* $a\left(\dfrac{x}{k}\right)^2 + b\left(\dfrac{x}{k}\right) + c = 0$ *i.e.,* $\boldsymbol{ax^2 + kbx + k^2c = 0}$

14. Relation between the roots of a cubic equation and its coefficients.

Let the cubic equation be $x^3 + S_1x^2 + S_2x + S_3 = 0$, where S_1, S_2, S_3 are the coefficients.

Let α, β, γ be the roots of the given cubic equation. Then,

$$\boldsymbol{S_1 = -(\alpha + \beta + \gamma),\quad S_2 = (\alpha\beta + \beta\gamma + \gamma\alpha),\quad S_3 = -(\alpha\beta\gamma)}$$

Conversely, if the roots of a cubic equation are given as $\alpha_1, \beta_1, \gamma$, then its equation can be written as :

$\boldsymbol{x^3 - S_1x^2 + S_2x - S_3 = 0,}$ where

$S_1 = (\alpha + \beta + \gamma), S_2 = (\alpha\beta + \beta\gamma + \gamma\alpha)$ and $S_3 = \alpha\beta\gamma$.

15. Relation between the roots of a bi-quadratic equation (degree 4) and its coefficients.

A bi-quadratic equation, whose roots are α, β, γ and δ is

$$x^4 - S_1 x^3 + S_2 x^2 - S_2x + S_4 = 0$$

where $S_1 = \alpha + \beta + \gamma + \delta,\ S_2 = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta,\ S_3 = \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta_L,\ S_4 = \alpha\beta\gamma\delta.$

SOLVED EXAMPLES

Ex. 1. What are the roots of the equation $(a + b + x)^{-1} = a^{-1} + b^{-1} + x^{-1}$? ***(CDS 2007)***

Sol. Given, $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x} \Rightarrow \dfrac{1}{a+b+x} - \dfrac{1}{x} = \dfrac{1}{a} + \dfrac{1}{b}$

$\Rightarrow \dfrac{x-(a+b+x)}{x(a+b+x)} = \dfrac{a+b}{ab} \Rightarrow \dfrac{-(a+b)}{x(a+b+x)} = \dfrac{a+b}{ab} \Rightarrow -ab = x^2 + (a+b)x$

$\Rightarrow x^2 + (a + b)x + ab = 0 \Rightarrow (x + a)(x + b) = 0 \Rightarrow \boldsymbol{x = -a, -b.}$

Ex. 2. What is one of the roots of the equation $\sqrt{\dfrac{2x}{3-x}} - \sqrt{\dfrac{3-x}{2x}} = \dfrac{3}{2}$?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4 ***(CDS 2008)***

Sol. Given equation is $\sqrt{\dfrac{2x}{3-x}} - \sqrt{\dfrac{3-x}{2x}} = \dfrac{3}{2}$

Let $\sqrt{\dfrac{2x}{3-x}} = a$. Then, the given equation reduces to $a - \dfrac{1}{a} = \dfrac{3}{2}$

$\Rightarrow\ 2(a^2 - 1) = 3a \quad \Rightarrow \quad 2a^2 - 3a - 2 = 0 \quad \Rightarrow \quad 2a^2 - 4a + a - 2 = 0$

$\Rightarrow\ 2a\,(a - 2) + 1\,(a - 2) = 0 \quad \Rightarrow \quad (2a + 1)\,(a - 2) = 0$

$\Rightarrow\ a - 2 = 0 \quad \Rightarrow\ a = 2$ or $(2a + 1) = 0 \ \Rightarrow\ a = -\dfrac{1}{2}$

$\therefore\ \sqrt{\dfrac{2x}{3-x}} = 2 \quad \Rightarrow\ 2x = 4(3 - x)$

$\Rightarrow 6x = 12 \ \Rightarrow\ x = 2$

or

$\sqrt{\dfrac{2x}{3-x}} = -\dfrac{1}{2} \quad \Rightarrow \quad \dfrac{2x}{3-x} = \dfrac{1}{4}$

$\Rightarrow 8x = 3 - x \quad \Rightarrow\ 9x = 3 \Rightarrow x = \dfrac{1}{3}.$

Hence, according to the given options, (*b*) is correct.

Ex. 3. If $3^x + 27\,(3^{-x}) = 12$, then what is the value of x ? ***(CDS 2009)***

Sol. Given, $3^x + 27\,(3^{-x}) = 12$

Let $3^x = y$. Then, $y + \dfrac{27}{y} = 12 \quad \Rightarrow\ y^2 - 12y + 27 = 0$

$\Rightarrow y^2 - 9y - 3y + 27 = 0 \quad \Rightarrow\ (y - 3)\,(y - 9) = 0 \quad \Rightarrow\ y = 3, 9$

$\Rightarrow 3^x = 3$ or $3^x = 9 \quad \Rightarrow\ x =$ **1 or 2.**

Ex. 4. What is the ratio of sum of squares of roots to the product of the roots of the equation $7x^2 + 12x + 18 = 0$? ***(CDS 2009)***

Sol. Let α, β be the roots of the equation $7x^2 + 12x + 18 = 0$.

$\left[\text{For a quadratic equation } ax^2 + bx + c = 0, \text{ sum of roots} = -\dfrac{b}{a}, \text{ product of roots} = +\dfrac{c}{a}\right]$

$\therefore\ \alpha + \beta = -\dfrac{12}{7}$ and $\alpha\beta = \dfrac{18}{7}$

$\Rightarrow (\alpha + \beta)^2 = \left(\dfrac{-12}{7}\right)^2 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = \dfrac{144}{49}$

$\Rightarrow\ \alpha^2 + \beta^2 = \dfrac{144}{49} - \dfrac{36}{7} = \dfrac{-108}{49}$

$\therefore$ Required ratio $= \alpha^2 + \beta^2 : \alpha\beta = \dfrac{\frac{-108}{49}}{18/7} = -\dfrac{6}{7} = -$ **6 : 7.**

Ex. 5. What is the value of a for which the equation $2x^2 + 2\sqrt{6}\,x + a = 0$ has equal roots ? ***(Kerala PET 2010)***

Sol. The equation $2x^2 + 2\sqrt{6}x + a = 0$ has equal roots if the discriminant $D = 0$.

$\therefore$ Here, $D = (2\sqrt{6})^2 - 4 \times (2) \times (a) = 0$ $\qquad [D = b^2 - 4ac$ for $ax^2 + bx + c = 0]$

$\Rightarrow 24 - 8a = 0 \Rightarrow a =$ **3.**

Ex. 6. Of the following quadratic equations, which is the one whose roots are 2 and – 15 ?

(*a*) $x^2 - 2x + 15 = 0$ (*b*) $x^2 + 15x - 2 = 0$ (*c*) $x^2 + 13x - 30 = 0$ (*d*) $x^2 - 30 = 0$. ***(MAT)***

Sol. Sum of roots = 2 + (– 15) = – 13; Product of roots = 2 × (– 15) = – 30.

$\therefore$ Required equation is x^2 – (sum of roots) x + product of roots = 0

$\Rightarrow$ Reqd. equation = $\mathbf{x^2 - 13x - 30 = 0.}$

Ex. 7. If one of the roots of the equation $x^2 + ax + 3 = 0$ is 3 and one of the roots of the equation $x^2 + ax + b = 0$ is three times the other root, then what is the value of b ? ***(J&K CET 2005)***

Sol. Let 3 and α be the roots of the equation $x^2 + ax + 3 = 0$

Then, sum of roots $= 3 + \alpha = -a$ ...(*i*), Product of roots $= 3\alpha = 3$ ...(*ii*)

From (*ii*) $\alpha = 1$. $\therefore$ Substituting $\alpha = 1$ in (*i*), we get $a = -4$.

$\therefore$ The second equation $x^2 + ax + b = 0$ becomes $x^2 - 4x + b = 0$.

Let β and 3β be the roots of this equation. Then, sum of roots $= \beta + 3\beta = 4 \Rightarrow 4\beta = 4 \Rightarrow \beta = 1$

and Product of roots $= \beta \times 3\beta = b \Rightarrow 3\beta^2 = b \Rightarrow \boldsymbol{b = 3.}$

Ex. 8. If α, β be the two roots of the equation $x^2 + x + 1 = 0$, then the equation whose roots are α/β and β/α is ?

(*a*) $x^2 - x - 1 = 0$ (*b*) $x^2 - x + 1 = 0$ (*c*) $x^2 + x - 1 = 0$ (*d*) $x^2 + x + 1 = 0$

(UPSEE 2005)

Sol. Let α, β be the roots of the equations $x^2 + x + 1 = 0$. Then,

Sum of roots $= \alpha + \beta = -1$, Product of roots $= \alpha\beta = 1$

Now the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$$x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \left(\frac{\alpha}{\beta} \times \frac{\beta}{\alpha}\right) = 0.$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{(-1)^2 - 2(1)}{1} = -1 \text{ and } \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1.$$

$\therefore$ Required equation $\boldsymbol{= x^2 + x + 1 = 0.}$

Ex. 9. If the difference in the roots of the equation $x^2 - px + q = 0$ is unity, then which one of the following is correct ?

(*a*) $p^2 + 4q = 1$ (*b*) $p^2 - 4q = 1$ (*c*) $p^2 + 4q = -1$ (*d*) $p^2 - 4q = -1$.

(CDS 2005)

Sol. Given, $x^2 - px + q = 0$

Let α, β be the roots of the given equation. Then,

$$\alpha + \beta = -\frac{(-p)}{1} = p \quad ...(i), \qquad \alpha\beta = \frac{q}{1} = q \quad ...(ii)$$

Also, $\alpha - \beta = 1$ (given) ...(*iii*)

$\therefore$ From (*i*) and (*iii*), $2\alpha = p + 1 \Rightarrow \alpha = \frac{p+1}{2}$

$\therefore$ From (*i*) and (*iii*), $2\beta = p - 1 \Rightarrow \beta = \frac{p-1}{2}$

Substituting these values of α and β in (*ii*), we have $\left(\frac{p+1}{2}\right)\left(\frac{p-1}{2}\right) = q$

$\Rightarrow \frac{p^2 - 1}{4} = q \Rightarrow p^2 - 1 = 4q \Rightarrow \boldsymbol{p^2 - 4q = 1.}$

Ex. 10. If the roots of the equation $x^2 + x + 1 = 0$ are in the ratio of $m : n$, then which one of the following relation holds ?

(*a*) $m + n + 1 = 0$ (*b*) $\frac{m}{n} + \frac{n}{m} + 1 = 0$ (*c*) $\sqrt{m} + \sqrt{n} + 1 = 0$ (*d*) $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} + 1 = 0$.

(CDS 2005)

Sol. $x^2 + x + 1 = 0$

$\therefore$ Roots are $= \dfrac{-1 \pm \sqrt{1-4}}{2} = \dfrac{-1 \pm \sqrt{3}\,i}{2}$ (where $= i = \sqrt{-1}$) $\left[\because \text{Roots} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$.

Given, $\dfrac{\dfrac{-1+\sqrt{3}\,i}{2}}{\dfrac{-1-\sqrt{3}\,i}{2}} = \dfrac{m}{n} \Rightarrow \dfrac{-1+\sqrt{3}\,i}{-1-\sqrt{3}\,i} = \dfrac{m}{n}$

$\Rightarrow \dfrac{m+n}{m-n} = \dfrac{(-1+\sqrt{3}\,i)+(-1-\sqrt{3}\,i)}{(-1+\sqrt{3}\,i)-(-1-\sqrt{3}\,i)} = \dfrac{-2}{2\sqrt{3}\,i}$ (Applying componendo and dividendo)

$\Rightarrow \dfrac{m+n}{m-n} = \dfrac{-1}{\sqrt{3}\,i} = \dfrac{i^2}{\sqrt{3}\,i} = \dfrac{i}{\sqrt{3}}$ $(\because i^2 = -1)$

$\Rightarrow \left(\dfrac{m+n}{m-n}\right)^2 = \left(\dfrac{i}{\sqrt{3}}\right)^2 \Rightarrow \dfrac{m^2+n^2+2mn}{m^2+n^2-2mn} = \dfrac{-1}{3} \quad \Rightarrow \dfrac{(m^2+n^2+2mn)+(m^2+n^2-2mn)}{(m^2+n^2+2mn)-(m^2+n^2-2mn)} = \dfrac{-1+3}{-1-3}$

$\Rightarrow \dfrac{2(m^2+n^2)}{2\,(2mn)} = \dfrac{2}{-4} \Rightarrow \dfrac{m^2+n^2}{2mn} = \dfrac{1}{-2} \quad \Rightarrow \dfrac{m^2+n^2}{mn} = -1 \Rightarrow \boldsymbol{\dfrac{m}{n} + \dfrac{n}{m} + 1 = 0.}$

Ex. 11. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then which one of the following is correct ?

(*a*) $a < 2$ (*b*) $2 < a < 3$ (*c*) $3 < a < 4$ (*d*) $a > 4$ *(CDS 2012)*

Sol. If the roots of the equation $x^2 - 2ax + a^2 - a - 3 = 0$ are real and less than 3, then $D \geq 0$ and $f(3) > 0$.

$\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0$ and $(3)^2 - 2a\,(3) + a^2 + a - 3 > 0$

$\Rightarrow a^2 - a^2 - a + 3 \geq 0$ and $9 - 6a + a^2 + a - 3 > 0$

$\Rightarrow -a + 3 \geq 0$ and $a^2 - 5a + 6 > 0 \Rightarrow a - 3 \leq 0$ and $(a-2)\,(a-3) > 0$

$\Rightarrow a \leq 3$ and $a < 2$ or $a > 3 \quad \Rightarrow$ **$a < 2$.**

Ex. 12. What are the number of solutions for real x, which satisfy the equation

$$\mathbf{2 \log_2 \log_2 x + \log_{1/2} \log_2 (2\sqrt{2}x) = 1?}$$

Sol. $2 \log_2 \log_2 x + \log_{1/2} \log_2 (2\sqrt{2}x) = 1$

$\Rightarrow 2 \log_2 \log_2 x - \log_2 \log_2 (2\sqrt{2}x) = 1$ $(\because \log_{1/a} x = -\log_a x)$

$\Rightarrow \log_2 (\log_2 x)^2 - \log_2 (\log_2 (2\sqrt{2}x)) = \log_2 2$

$\Rightarrow \log_2 \dfrac{(\log_2 x)^2}{\log_2(2\sqrt{2}x)} = \log_2 2 \Rightarrow \dfrac{(\log_2 x)^2}{\log_2(2\sqrt{2}x)} = 2 \Rightarrow (\log_2 x)^2 = 2 \log_2 (2\sqrt{2}x)$

$\Rightarrow (\log_2 x)^2 = 2 \log_2 (2^{3/2} x) \quad \Rightarrow (\log_2 x)^2 = 2\left[\dfrac{3}{2} \log 2x\right] = 2[3/2\ \{\log_2 2 + \log_2 x\}]$

$\Rightarrow (\log_2 x)^2 = 3 + 2 \log_2 x \quad \Rightarrow (\log_2 x)^2 - 2 \log_2 x - 3 = 0$

$\Rightarrow (\log_2 x - 3)\,(\log_2 x + 1) = 0 \Rightarrow \log_2 x = 3$ or $\log_2 x = -1 \quad \Rightarrow x = 2^3 = 8$ or $x = 2^{-1} = \dfrac{1}{2}$

But for $x = \dfrac{1}{2}$, $\log_2 \log_2 \left(\dfrac{1}{2}\right)$ is undefined so $\boldsymbol{x = 8}$ is the only possible value of x.

Ex. 13. If α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, then what is $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is equal to ?

Sol. Given α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$. Then,

$$S_1 = \alpha + \beta + \gamma = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3} = -a; \quad S_2 = \alpha\beta + \beta\gamma + \alpha\gamma = \frac{\text{coefficient of } x}{\text{coefficient of } x^3} = b$$

$$S_3 = \alpha\beta\gamma = \frac{-\text{ constant term}}{\text{coefficient of } x^3} = -c$$

$$\therefore\ \alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{S_2}{S_3} = \frac{b}{-c} = -\boldsymbol{\frac{b}{c}}.$$

Ex. 14. If α, β are the roots of the equation $9x^2 + 6x + 1 = 0$, then write the equation with roots $\frac{1}{\alpha}, \frac{1}{\beta}$.

Sol. As α, β are the roots of the equation $9x^2 + 6x + 1 = 0$, $\alpha + \beta = -\frac{6}{9} = -\frac{2}{3}$, $\alpha\beta = \frac{1}{9}$

$\therefore$ Required equation = x^2 – (Sum of roots)x + Product of roots = 0

$$i.e.,\ x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha}.\frac{1}{\beta} = 0,\ \ i.e.,\ x^2 - \left(\frac{\beta + \alpha}{\gamma\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

$$i.e.,\ x^2 - \left(\frac{-2/3}{1/9}\right)x + 9 = 0,\ \ i.e.,\ \mathbf{x^2 + 6x + 9 = 0}$$

Alternatively,

The equation whose roots $\left(\frac{1}{\alpha}, \frac{1}{\beta}\right)$ are the reciprocals of the roots (α, β) of the equation $9x^2 + 6x + 1 = 0$ can be obtained by replacing x by $\frac{1}{x}$ in the given equation.

$\therefore$ Required equation is : $9\left(\frac{1}{x}\right)^2 + 6\left(\frac{1}{x}\right) + 1 = 0 \ \Rightarrow\ \mathbf{x^2 + 6x + 9 = 0.}$

Ex. 15. Find the values of k for which the equations $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$ will have a common root?

Sol. Let α be the common root of both the given equations. Then α satisfies both the equations. So,

$$\alpha^2 - k\alpha - 21 = 0 \qquad ...(i)$$

$$\alpha^2 - 3k\alpha + 35 = 0 \qquad ...(ii)$$

Solving equations (i) and (ii) simultaneously, we get

$$\frac{\alpha^2}{-35k - 63k} = \frac{\alpha}{-21 - 35} = \frac{1}{-3k + k}$$

$$\left[\begin{array}{c} a_1x + b_1y + c_1 = 0;\ a_2x + b_2y + c_2 = 0 \\ \dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{1}{a_1b_2 - a_2b_1} \end{array}\right]$$

$$\Rightarrow \alpha^2 = \frac{-98k}{-2k} = 49 \text{ and } \alpha = \frac{-56}{-2k} = \frac{28}{k}$$

$$\therefore\ 49 = \left(\frac{28}{k}\right)^2 \Rightarrow k^2 = \frac{28 \times 28}{49} = 16 \Rightarrow k = \boldsymbol{\pm 4}.$$

PRACTICE SHEET

1. What are the roots of the equation $\log_{10}(x^2 - 6x + 45) = 2$?

(a) 9, –5 (b) –9, 5 (c) 11, – 5 (d) –11, 5

(*CDS 2010*)

2. What is one of the values of x in the equation $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6}$

(a) $\frac{5}{13}$ (b) $\frac{7}{13}$ (c) $\frac{9}{13}$ (d) $\frac{11}{3}$

(*CDS 2007*)

3. What are the roots of the equation $4^x - 3.2^{x+2} + 32 = 0$?

(a) 1, 2 (b) 3, 4 (c) 2, 3 (d) 1, 3

(*CDS 2010*)

4. What are the roots of the quadratic equation $a^2 b^2 x^2 - (a^2 + b^2)x + 1 = 0$?

(*a*) $\frac{1}{a^2}, \frac{1}{b^2}$ (*b*) $-\frac{1}{a^2}, -\frac{1}{b^2}$

(*c*) $\frac{1}{a^2}, -\frac{1}{b^2}$ (*d*) $-\frac{1}{a^2}, \frac{1}{b^2}$ (***CDS 2011***)

5. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ for $a \neq 0$ are real and equal, then the value of $a^3 + b^3 + c^3$ is :

(*a*) abc (*b*) $3abc$

(*c*) 0 (*d*) None of these

(***MAT 2003***)

6. If sin θ and cos θ are the roots of the equations $ax^2 - bx + c = 0$, then which of the following is correct?

(*a*) $a^2 + b^2 + 2ac = 0$ (*b*) $a^2 - b^2 + 2ac = 0$

(*c*) $a^2 + b^2 + 2ab = 0$ (*d*) $a^2 - b^2 - 2ac = 0$.

(***CDS 2011***)

7. The roots of the quadratic equation $x^2 - 2\sqrt{3}x - 22 = 0$ are:

(*a*) imaginary (*b*) real, rational, equal

(*c*) real, rational, unequal (*d*) real, irrational, unequal

(***WBJEE 2010***)

8. Which one of the following is the equation whose roots are respectively three times the roots of the equation $ax^2 + bx + c = 0$?

(*a*) $ax^2 + 3bx + c = 0$ (*b*) $ax^2 + 3bx + 9c = 0$

(*c*) $ax^2 - 3bx + 9c = 0$ (*d*) $ax^2 + bx + 3c = 0$

(***CDS 2007***)

9. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\alpha\beta^2 + \alpha^2\beta + \alpha\beta$ equals :

(*a*) $\frac{bc}{-a^2}$ (*b*) 0 (*c*) abc (*d*) $\frac{c(a-b)}{a^2}$

(***AMU 2000***)

10. For what value of m the ratio of the roots of the equation $12x^2 - mx + 5 = 0$ is 3 : 2 ?

(*a*) $5\sqrt{10}$ (*b*) $10\sqrt{5}$ (*c*) $25\sqrt{2}$ (*d*) $15\sqrt{5}$

(***Rajasthan PET 2002***)

11. If the roots of the equation $ax^2 + bx + c = 0$ are equal in magnitude but opposite in sign, then which one of the following is correct ?

(*a*) $a = 0$ (*b*) $b = 0$

(*c*) $c = 0$ (*d*) $b = 0, c \neq 0, a \neq 0$.

(***CDS 2005***)

12. If $2x^2 - 7xy + 3y^2 = 0$, then the value of $x : y$ is

(*a*) 3 : 2 (*b*) 2 : 3

(*c*) 3 : 1 and 1 : 2 (*d*) 5 : 6

(***MAT 2003***)

13. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, such that $\beta = \alpha^{1/3}$, then

(*a*) $(a^3 b)^{1/4} + (ac^3)^{1/4} + a = 0$

(*b*) $(a^3 c)^{1/4} + (ac^3)^{1/4} + b = 0$

(*c*) $(a^3 b)^{1/4} + (ab^3)^{1/4} + c = 0$

(*d*) $(b^3 c)^{1/4} + (bc^3)^{1/4} + a = 0$ (***Kerala PET 2003***)

14. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then a, b, c are in :

(*a*) AP (*b*) GP

(*c*) HP (*d*) None of these

15. If an integer P is chosen at random in the interval $0 \leq p \leq 5$, the probability that the roots of the equation $x^2 + px + \frac{p}{4} + \frac{1}{2} = 0$ are real is

(*a*) $\frac{2}{3}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{4}{5}$

16. Two students A and B solve an equation of the form $x^2 + px + q = 0$. A starts with a wrong value of p and obtains the roots as 2 and 6. B starts with a wrong value of q and gets the roots as 2 and –9. What are the correct roots of the equations ?

(*a*) 3 and –4 (*b*) –3 and –4 (*c*) –3 and 4 (*d*) 3 and 4

(***CDS 2012***)

17. If α and β are the roots of the equation $x^2 - 6x + 6 = 0$, what is $\alpha^3 + \beta^3 + \alpha^2 + \beta^2 + \alpha + \beta$ equal to ?

(*a*) 150 (*b*) 138 (*c*) 128 (*d*) 124

(***CDS 2011***)

18. If α and β are the roots of the equation $x^2 + px + q = 0$, then $-\alpha^{-1}$ and $-\beta^{-1}$ are the roots of which one of the following equations ?

(*a*) $qx^2 - px + 1 = 0$ (*b*) $q^2 + px + 1 = 0$

(*c*) $x^2 + px - q = 0$ (*d*) $x^2 - px + q = 0$

(***CDS 2010***)

19. The number of solution of $\log_4 (x - 1) = \log_2 (x - 3)$ is :

(*a*) 0 (*b*) 5 (*c*) 2 (*d*) 3

(***AMU 2007***)

20. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(*a*) no real roots (*b*) exactly one real root

(*c*) exactly four real roots (*d*) infinite number of real roots.

(***AIEEE 2012***)

21. If $5^{56}\left(\frac{1}{5}\right)^{x}\left(\frac{1}{5}\right)^{\sqrt{x}} > 1$, then x satisfies :

(*a*) [0, 49) (*b*) (49, 64] (*c*) [0, 64) (*d*) [49, 64)

(***DCE 2007***)

22. The sum of the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero. What is the product of the roots of the equation ?

(*a*) $-\frac{(a+b)}{2}$ (*b*) $\frac{(a+b)}{2}$

(*c*) $-\frac{(a^2+b^2)}{2}$ (*d*) $\frac{(a^2+b^2)}{2}$ (***CDS 2010***)

23. For what value of k will the roots of the equation $kx^2 - 5x + 6 = 0$ be in the ratio 2 : 3 ?

(*a*) 0 (*b*) 1 (*c*) –1 (*d*) 2

(***CDS 2010***)

24. The number of real solutions of the equation $2|x|^2 - 5|x| + 2 = 0$ is :

(*a*) 0 (*b*) 4

(*c*) 2 (*d*) None of these

25. If p, q, r are positive and are in A.P., the roots of quadratic equation $px^2 + qx + r = 0$ are real for :

(*a*) $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ (*b*) $\left|\frac{p}{r} - 7\right| \geq 4\sqrt{3}$

(*c*) all p and r (*d*) no p and r

26. The values of x which satisfy the expression $(5 + 2\sqrt{6})^{x^2+3} + (5 - 2\sqrt{6})^{x^2-3} = 10$ are :

(*a*) $\pm 2, \pm\sqrt{3}$ (*b*) $\pm\sqrt{2}, \pm 4$ (*c*) $\pm 2, \pm\sqrt{2}$ (*d*) $2, \sqrt{2}, \sqrt{3}$

27. If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

(*a*) $\frac{-15}{4}$ (*b*) $\frac{-9}{4}$ (*c*) $\frac{13}{4}$ (*d*) 4

(***KCET 2005***)

28. If α, β, γ are the roots of the equation $x^3 + 4x + 1 = 0$, then $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1}$ is equal to

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

(***UPSEE 2003***)

29. If the roots of $x^3 - 12x^2 + 12x - 28 = 0$ are in A.P, their common difference is

(*a*) ± 3 (*b*) ± 2

(*c*) ± 1 (*d*) None of these

(***Rajasthan PET 2001***)

30. The quadratic equation whose roots are three times the roots of $3ax^2 + 3bx + c = 0$ is

(*a*) $ax^2 + bx + 3c = 0$ (*b*) $ax^2 + 3bx + c = 0$

(*c*) $ax^2 + 3bx + 3c = 0$ (*d*) $9ax^2 + 9bx + c = 0$.

(***WBJEE 2009***)

31. If α and β are the roots of the equation $ax^2 + bx + c = 0$ and if $px^2 + qx + r = 0$ has roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$, then r equals:

(*a*) abc (*b*) $a + 2b$ (*c*) $a + b + c$ (*d*) $ab + bc + ca$.

32. The equation whose roots are the negatives of the roots of the equation $x^7 + 3x^5 + x^3 - x^2 + 7x + 2 = 0$ is :

(*a*) $x^7 + 3x^5 + x^3 - x^2 - 7x - 2 = 0$

(*b*) $x^7 + 3x^5 + x^3 - x^2 + 7x - 2 = 0$

(*c*) $x^7 + 3x^5 + x^3 + x^2 - 7x + 2 = 0$

(*d*) $x^7 + 3x^5 + x^3 + x^2 + 7x - 2 = 0$ (***EAMCET 2001***)

33. Given that α, γ are the roots of the equation $Ax^2 - 4x + 1 = 0$ and β, δ are the roots of the equation $Bx^2 - 6x + 1 = 0$, then the values of A and B respectively such that α, β, γ and δ are in H.P are :

(*a*) –5, 9 (*b*) 3/2, 5

(*c*) 3, 8 (*d*) None of these

34. Let α, β be the roots of the equation $(x - a)(x - b) = c, c \neq 0$, then the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are :

(*a*) a, c (*b*) b, c (*c*) a, b (*d*) $a + c, b + c$

35. If the roots of the equation $x^3 - ax^2 + bx - c = 0$ are three consecutive integers, then what is the smallest possible value of b ?

(*a*) $-\frac{1}{\sqrt{3}}$ (*b*) –1 (*c*) 0 (*d*) 1 (***CAT***)

36. If two equations $x^2 + a^2 = 1 - 2ax$ and $x^2 + b^2 = 1 - 2bx$ have only one common root, then

(*a*) $(a - b) = -1$ (*b*) $|a - b| = 1$

(*c*) $a - b = 1$ (*d*) $|a - b| = 2$ (***DCE 2004***)

37. If α, β, γ are the roots of the equation $x^3 - 3x + 11 = 0$, then the equation whose roots are $(\alpha + \beta), (\beta + \gamma), (\gamma + \alpha)$ is :

(*a*) $x^3 + 3x + 11 = 0$ (*b*) $x^3 - 3x + 11 = 0$

(*c*) $x^3 + 3x - 11 = 0$ (*d*) $x^3 - 3x - 11 = 0$

38. If α, β are the roots of $ax^2 + bx + c = 0$, and $\alpha + k, \beta + k$ are the roots of $px^2 + qx + r = 0$, then $k =$

(*a*) $-\frac{1}{2}(a/b - p/q)$ (*b*) $(a/b - p/q)$

(*c*) $\frac{1}{2}(b/a - q/p)$ (*d*) $(ab - pq)$

39. Find the value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\infty}}}$

(*a*) –4 (*b*) 2 (*c*) 3 (*d*) 6

40. The roots of $(x - a)(x - a - 1) + (x - a - 1)(x - a - 2) + (x - a)(x - a - 2) = 0, a \in R$ are always :

(*a*) imaginary (*b*) real and distinct

(*c*) equal (*d*) rational and equal

ANSWERS

1. (*c*)	**2.** (*c*)	**3.** (*c*)	**4.** (*a*)	**5.** (*b*)	**6.** (*b*)	**7.** (*c*)	**8.** (*b*)	**9.** (*d*)	**10.** (*a*)
11. (*d*)	**12.** (*c*)	**13.** (*b*)	**14.** (*c*)	**15.** (*a*)	**16.** (*b*)	**17.** (*b*)	**18.** (*a*)	**19.** (*b*)	**20.** (*a*)
21. (*a*)	**22.** (*c*)	**23.** (*b*)	**24.** (*b*)	**25.** (*b*)	**26.** (*c*)	**27.** (*a*)	**28.** (*c*)	**29.** (*a*)	**30.** (*c*)
31. (*c*)	**32.** (*d*)	**33.** (*c*)	**34.** (*c*)	**35.** (*b*)	**36.** (*d*)	**37.** (*d*)	**38.** (*c*)	**39.** (*c*)	**40.** (*b*)

HINTS AND SOLUTIONS

1. Given, $\log_{10}(x^2 - 6x + 45) = 2 \Rightarrow x^2 - 6x + 45 = 10^2 = 100$

$\Rightarrow x^2 - 6x - 55 = 0 \Rightarrow x^2 - 11x + 5x - 55 = 0$

$\Rightarrow x(x - 11) + 5(x - 11) = 0 \Rightarrow (x + 5)(x - 11) = 0$

$\Rightarrow x = \mathbf{-5 \text{ or } 11.}$

2. Let $\sqrt{\frac{x}{1-x}} = y$. Then, the given equation reduces to

$$y + \frac{1}{y} = \frac{13}{6} \Rightarrow 6(y^2 + 1) = 13y$$

$\Rightarrow 6y^2 - 13y + 6 = 0 \Rightarrow 6y^2 - 9y - 4y + 6 = 0$

$\Rightarrow 3y(2y - 3) - 2(2y - 3) = 0$

$\Rightarrow (3y - 2)(2y - 3) = 0 \quad \Rightarrow y = \frac{2}{3}$ and $\frac{3}{2}$

when $y = \frac{2}{3}, \sqrt{\frac{x}{1-x}} = \frac{2}{3} \Rightarrow \frac{x}{1-x} = \frac{4}{9}$

$\Rightarrow 9x = 4 - 4x \Rightarrow 13x = 4 \Rightarrow x = \mathbf{\frac{4}{13}}$

when $y = \frac{3}{2}, \sqrt{\frac{x}{1-x}} = \frac{3}{2} \Rightarrow \frac{x}{1-x} = \frac{9}{4}$

$\Rightarrow 4x = 9 - 9x \Rightarrow 13x = 9 \Rightarrow x = \mathbf{\frac{9}{13}.}$

3. $4^x - 3.2^{x+2} + 32 = 0$

$\Rightarrow 2^{2x} - 3.2^2.2^x + 32 = 0 \quad \Rightarrow 2^{2x} - 12.2^x + 32 = 0$

Let $2^x = a$. Then, $a^2 - 12a + 32 = 0$

$\Rightarrow (a - 8)(a - 4) = 0 \Rightarrow a = 8$ and 4

$\Rightarrow 2^x = 8$ and $2^x = 4 \quad \Rightarrow \boldsymbol{x = 3}$ **and** $\boldsymbol{x = 2.}$

4. Let the roots of the equation $a^2b^2x^2 - (a^2 + b^2)x + 1 = 0$ be α and β. Then,

$$\alpha + \beta = \frac{a^2 + b^2}{a^2b^2} \quad ....(i), \quad \alpha\beta = \frac{1}{a^2b^2} \quad ...(ii)$$

$$\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \sqrt{\left(\frac{a^2 + b^2}{a^2b^2}\right)^2 - \frac{4}{a^2b^2}}$$

$$= \sqrt{\frac{a^4 + b^4 + 2a^2b^2 - 4a^2b^2}{(a^2b^2)^2}}$$

$$= \sqrt{\frac{(a^2 - b^2)^2}{(a^2b^2)^2}} = \frac{a^2 - b^2}{a^2b^2} \quad ...(iii)$$

$\therefore$ On solving (i) and (ii), we get $\alpha = \mathbf{\frac{1}{b^2}, \beta = \frac{1}{a^2}.}$

5. Given that the roots are real and equal,

$D = 0 \Rightarrow b^2 - 4ac = 0$ for $ax^2 + bx + c = 0$.

$\therefore [-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$\Rightarrow 4[a^4 + b^2c^2 - 2a^2bc - c^2b^2 + ac^3 + ab^3 - a^2bc) = 0$

$\Rightarrow 4a(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow \boldsymbol{a^3 + b^3 + c^3 = 3abc.}$

6. As $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$.

$\therefore \sin\theta + \cos\theta = \frac{b}{a}$ and $\sin\theta\cos\theta = \frac{c}{a}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = \frac{b^2}{a^2}$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2}$

$\Rightarrow 1 + \frac{2c}{a} = \frac{b^2}{a^2} \Rightarrow \frac{2c}{a} = \frac{b^2}{a^2} - 1 = \frac{b^2 - a^2}{a^2}$

$\Rightarrow 2ac = b^2 - a^2 \Rightarrow \boldsymbol{a^2 - b^2 + 2ac = 0.}$

7. Given equation is $x^2 - 2\sqrt{3}x - 22 = 0$.

Discriminant $= D = b^2 - 4ac$ (for $ax^2 + bx + c = 0$)

$$= (-2\sqrt{3})^2 - 4(-22) = 12 + 88 = 100$$

As $D > 0$ and is a perfect square, the roots are real, rational and unequal.

8. Let α, β be the roots of the equation $ax^2 + bx + c = 0$. Then,

$$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}.$$

By the given condition, roots of the required equation are 3α and 3β.

$\therefore$ Sum of roots $= 3\alpha + 3\beta = 3(\alpha + \beta) = -\frac{3b}{a}$

Product of roots $= 3\alpha \,.\, 3\beta = 9\alpha\beta = \frac{9c}{a}$

$\therefore$ Required equation

$= x^2 -$ (sum of roots)x + (product of roots) $= 0$

$\Rightarrow x^2 - \left(\frac{-3b}{a}x\right) + \frac{9c}{a} = 0$

$\Rightarrow \boldsymbol{ax^2 + 3bx + 9c = 0.}$

9. Given, α, β are the roots of the equation $ax^2 + bx + c = 0$. Then, $\alpha + \beta = -b/a$, $\alpha\beta = c/a$

Then, $\alpha\beta^2 + \alpha^2\beta + \alpha\beta = \alpha\beta(\alpha + \beta) + \alpha\beta = (c/a)(-b/a) + c/a$

$$= -\frac{bc}{a^2} + \frac{c}{a} = \frac{-bc + ac}{a^2} = \mathbf{\frac{c(a - b)}{a^2}.}$$

10. Given, the roots of the given equation $12x^2 - mx + 5 = 0$ are in the ratio $3 : 2$. Let the roots of the given equation be 3α and 2α. Then,

Sum of roots $= 3\alpha + 2\alpha = \frac{m}{12} \Rightarrow 5\alpha = \frac{m}{12}$ ...(i)

and $(3\alpha)(2\alpha) = \frac{5}{12} \Rightarrow 6\alpha^2 = \frac{5}{12} \Rightarrow \alpha^2 = \frac{5}{72}$

$\Rightarrow \alpha = \sqrt{\frac{5}{72}}$ ...(ii)

$\therefore$ From (*i*) and (*ii*)

$$5.\sqrt{\frac{5}{72}} = \frac{m}{12} \Rightarrow m = 60\sqrt{\frac{5}{72}} = 60.\frac{\sqrt{5}}{6\sqrt{2}} = 10\sqrt{\frac{5}{2}}$$

$$= 10\cdot\frac{\sqrt{5}}{\sqrt{2}}.\frac{\sqrt{2}}{\sqrt{2}} = \frac{10}{2}.\sqrt{10} = \mathbf{5\sqrt{10}}.$$

11. Given equation is $ax^2 + bx + c = 0$.

$\therefore$ Roots are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Given $\left[\dfrac{-b+\sqrt{b^2-4ac}}{2a}\right] = -\left[\dfrac{-b-\sqrt{b^2-4ac}}{2a}\right]$

$\Rightarrow -b + \sqrt{b^2 - 4ac} = b + \sqrt{b^2 - 4ac}$

$\Rightarrow 2b = 0 \Rightarrow b = 0$. but $a \neq 0$, $c \neq 0$.

12. $2x^2 - 7xy + 3y^2 = 0$

$\Rightarrow 2\left(\dfrac{x}{y}\right)^2 - 7\left(\dfrac{x}{y}\right) + 3 = 0$

$\therefore \dfrac{x}{y} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{7 \pm \sqrt{49 - 24}}{4} = \dfrac{7 \pm 5}{4} = 3, \dfrac{1}{2}$

$\therefore$ $x : y =$ **3 : 1 and 1 : 2.**

13. Let α, β be the roots of the equation $ax^2 + bx + c = 0$. Then,

$\alpha + \beta = -\dfrac{b}{a}$ ...(*i*)

$\alpha\beta = \dfrac{c}{a}$ ...(*ii*)

and $\beta = \alpha^{1/3}$ ...(*iii*)

$\therefore$ From (*ii*) and (*iii*), $\alpha.(\alpha)^{1/3} = \dfrac{c}{a} \Rightarrow \alpha^{4/3} = \dfrac{c}{a} \Rightarrow \alpha = \left(\dfrac{c}{a}\right)^{3/4}$

$\therefore \quad \beta = \left(\left(\dfrac{c}{a}\right)^{3/4}\right)^{1/3} = \left(\dfrac{c}{a}\right)^{1/4}$

$\therefore$ Putting these values of α and β in eqn. (*i*), we have

$\left(\dfrac{c}{a}\right)^{3/4} + (c/a)^{1/4} = -\dfrac{b}{a}$

$\Rightarrow a.\, a^{-3/4}\, c^{3/4} + a.\, a^{-1/4}\, c^{1/4} = -b$

$\Rightarrow a^{1/4}\, c^{3/4} + a^{3/4}\, c^{1/4} + b = 0$

$\Rightarrow (ac^3)^{1/4} + (a^3\, c)^{1/4} + b = 0.$

14. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then Discriminant (D) = 0, *i.e.*,

$\Rightarrow b^2\,(c - a)^2 - 4a(b - c)\, c(a - b) = 0.$

$\Rightarrow b^2\,(c^2 + a^2 - 2ac) - 4ac\,(ab - ca - b^2 + bc) = 0$

$\Rightarrow b^2c^2 + b^2\,a^2 - 2ab^2c - 4a^2bc + 4a^2c^2 + 4ab^2c - 4abc^2 = 0$

$\Rightarrow a^2b^2 + b^2c^2 + 4a^2c^2 + 2ab^2c - 4a^2bc - 4abc^2 = 0$

$\Rightarrow (ab + bc - 2ac)^2 = 0 \Rightarrow ab + bc - 2ac = 0$

$\Rightarrow ab + bc = 2ac \Rightarrow \dfrac{1}{c} + \dfrac{1}{a} = \dfrac{2}{b} \Rightarrow a, b, c$ are in H.P.

15. The equation $x^2 + px + \dfrac{p}{4} + \dfrac{1}{2} = 0$ has real roots if the discriminant $D \geq 0$.

$\Rightarrow p^2 - 4\left(\dfrac{p}{4} + \dfrac{1}{2}\right) \geq 0 \Rightarrow p^2 - p - 2 \geq 0$

$\Rightarrow p^2 - 2p + p - 2 \geq 0 \Rightarrow p(p - 2) + 1\,(p - 2) \geq 0$

$\Rightarrow (p - 2)\,(p + 1) \geq 0$

$\Rightarrow (p - 2) \geq 0$ and $(p + 1) \geq 0$

$\Rightarrow p \geq 2$ or $p \leq -1$

The condition $p \leq -1$ is not admissible as $0 \leq p \leq 5$.

Now $p \geq 2 \Rightarrow p$ can take up the value 2 or 3 or 4 or 5 from the given values. {0, 1, 2, 3, 4, 5}

$\therefore$ Probability (Roots of given equation are real)

$$= \frac{\text{Number of values } p \text{ can take}}{\text{Given number of values}} = \frac{4}{6} = \mathbf{\frac{2}{3}}.$$

16. Let the roots of the quadratic equation $x^2 + px + q = 0$ be α and β. According to the given condition, A starts with a wrong value of p and obtains the roots as 2 and 6. But this time, the value of q is correct.

$\therefore \quad q$ = Product of roots = $\alpha\beta = 2 \times 6 = 12$.

According to the second condition, B starts with a wrong value of q and obtains the roots as 2 and –9. But this time, the value of p is correct.

$\therefore$ p = sum of roots = $\alpha + \beta = 2 + (-9) = -7$ ...(*i*)

$\therefore (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = (-7)^2 - 4.12 = 49 - 48 = 1$

$\Rightarrow \alpha - \beta = 1$ ...(*ii*)

$\therefore$ Solving equations (*i*) and (*ii*), we get **$\alpha = -3$ and $\beta = -4$.**

17. $\alpha + \beta = 6$, $\alpha\beta = 6$

$\therefore (\alpha + \beta)^2 = 6^2 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 36$

$\Rightarrow \alpha^2 + \beta^2 = 36 - 2 \times 6 = 24$

Now, $\alpha^3 + \beta^3 + \alpha^2 + \beta^2 + \alpha + \beta$

$= (\alpha + \beta)\,(\alpha^2 + \beta^2 - \alpha\beta) + (\alpha^2 + \beta^2) + (\alpha + \beta)$

$= 6(24 - 6) + 24 + 6 = 6 \times 18 + 30 =$ **138.**

18. Since, α and β are the roots of the equation $x^2 + px + q = 0$,

$\therefore \alpha + \beta = -p$ and $\alpha\beta = q$

Now, equation whose roots are $-\alpha^{-1}$ and $-\beta^{-1}$ is

x^2 – (sum of the roots) x + product of the roots = 0

i.e., $x^2 - (-\alpha^{-1} - \beta^{-1})x + (-\alpha^{-1})\,(-\beta^{-1}) = 0$

$-\alpha^{-1} - \beta^{-1} = -\left(\dfrac{1}{\alpha} + \dfrac{1}{\beta}\right) = -\left(\dfrac{\alpha + \beta}{\alpha\beta}\right) = \dfrac{p}{q}$

and $(-\alpha^{-1})\,(-\beta^{-1}) = \dfrac{1}{\alpha\beta} = \dfrac{1}{q}$

$\therefore$ Required equation $= x^2 - \dfrac{p}{q}x + \dfrac{1}{q} = 0$

$\Rightarrow \boldsymbol{qx^2 - px + 1 = 0.}$

19. $\log_4 (x - 1) = \log_2 (x - 3)$

$\Rightarrow \log_{2^2} (x - 1) = \log_2 (x - 3)$

$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$$\left[\text{Using } \log_{m^n}(x) = \frac{1}{n}\log_m x\right]$$

$\Rightarrow \log_2(x-1) = 2\log_2(x-3)$

$\Rightarrow \log_2(x-1) = \log_2(x-3)^2$

$\Rightarrow (x-1) = (x-3)^2 \Rightarrow (x-1) = x^2 - 6x + 9$

$\Rightarrow x^2 - 7x + 10 = 0 \Rightarrow (x-2)(x-5) = 0 \Rightarrow x = 2$ or 5.

$x = 2$ is inadmissible as $\log_2(x-3)$ is not defined when $x = 2$.

$\therefore$ $\boldsymbol{x = 5.}$

20. Given, $e^{\sin x} - e^{-\sin x} - 4 = 0$

Let $e^{\sin x} = y$. Then, the given equation becomes

$$y - \frac{1}{y} = 4 \Rightarrow y^2 - 4y - 1 = 0$$

$$\Rightarrow \quad y = \frac{4 \pm \sqrt{16+4}}{2} = 2 \pm \sqrt{5}$$

$$\Rightarrow \quad e^{\sin x} = 2 \pm \sqrt{5} \Rightarrow \sin x = \log_e(2 \pm \sqrt{5})$$

$$\Rightarrow \quad \sin x = \log_e(2+\sqrt{5})$$

$(\because\ (2-\sqrt{5}) < 0$ and so $\log_e(2-\sqrt{5})$ is not defined)

Now $(2+\sqrt{5}) > 4 \Rightarrow \log_e(2+\sqrt{5}) > 1$

But the value of $\sin x$ lies between -1 and 1, both values inclusive, so $\sin x \neq \log_e(2+\sqrt{5})$

$\therefore$ There are no possible real roots of the given equation.

21. $5^{56}\left(\frac{1}{5}\right)^x\left(\frac{1}{5}\right)^{\sqrt{x}} > 1$

$\Rightarrow 5^{56} \times 5^{-x} \times 5^{-\sqrt{x}} > 1 \Rightarrow 5^{56-x-\sqrt{x}} > 5^0$

$\Rightarrow 56 - x - \sqrt{x} > 0 \Rightarrow x + \sqrt{x} - 56 < 0$

$\Rightarrow y^2 + y - 56 < 0$, where $y = \sqrt{x}$

$\Rightarrow (y+8)(y-7) < 0 \Rightarrow -8 < y < 7 \Rightarrow -8 < \sqrt{x} < 7$

$\Rightarrow 0 \leq \sqrt{x} < 7$ as $\sqrt{x}$ cannot be negative

$\Rightarrow 0 \leq x < 49 \Rightarrow x \in [0, 49)$

22. Given, $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$

$\Rightarrow \frac{(x+b)+(x+a)}{(x+a)(x+b)} = \frac{1}{c} \Rightarrow \frac{2x+b+a}{x^2+(a+b)x+ab} = \frac{1}{c}$

$\Rightarrow 2cx + (a+b)c = x^2 + (a+b)x + ab$

$\Rightarrow x^2 + (a+b-2c)x + ab - ac - bc = 0$

Let α, β be the roots of this equation. Then,

$\alpha + \beta = -(a+b-2c) = 0$ (Given)

$\Rightarrow a + b = 2c$ and $\alpha\beta = ab - ac - bc = ab - (a+b)c$

$$= ab - (a+b)\frac{(a+b)}{2}$$

$$= \frac{2ab - (a^2+b^2+2ab)}{2} = -\left(\boldsymbol{\frac{a^2+b^2}{2}}\right).$$

23. Let the roots of the equation $kx^2 - 5x + 6 = 0$ be α and β.

Then, $\alpha + \beta = 5/k$ ...(*i*)

$\alpha\beta = 6/k$ ...(*ii*)

Given $\alpha/\beta = \frac{2}{3} \Rightarrow \alpha = \frac{2}{3}\beta$

$\therefore$ From (*i*) and (*ii*),

$\frac{2}{3}\beta + \beta = \frac{5}{k}$ and $\frac{2}{3}\beta^2 = \frac{6}{k}$

$\Rightarrow \frac{5}{3}\beta = \frac{5}{k}$ and $\beta^2 = \frac{9}{k} \Rightarrow \beta = \frac{3}{k}$ and $\beta^2 = \frac{9}{k}$

$\Rightarrow \frac{9}{k^2} = \frac{9}{k} \Rightarrow 9k^2 - 9k = 0 \Rightarrow k(k-1) = 0 \Rightarrow k = 0$ or 1

But $k = 0$ does not satisfy the condition, so $\boldsymbol{k = 1}$.

24. $2|x|^2 - 5|x| + 2 = 0$

$\Rightarrow (2|x|-1)(|x|-2) = 0$

$\Rightarrow |x| = \frac{1}{2}, 2 \Rightarrow x = \pm\frac{1}{2}, \pm 2$

So, there are 4 solutions.

25. $\because$ p, q, r are in A.P.

$q = \frac{p+r}{2}$ $[\because p + r = 2q]$

For the real roots $q^2 - 4pr \geq 0$ [Disct. ≥ 0]

$\Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr \geq 0$

$\Rightarrow p^2 + r^2 - 14pr \geq 0$

$\Rightarrow \left(\frac{p}{r}\right)^2 - 14\left(\frac{p}{r}\right) + 1 \geq 0$

$\Rightarrow \left(\frac{p}{r} - 7\right)^2 \geq 48 = \boldsymbol{\left|\frac{p}{r} - 7\right| \geq 4\sqrt{3}.}$

26. Let $y = 5 + 2\sqrt{6}$. Then $\frac{1}{y} = 5 - 2\sqrt{6}$. Thus the given expression reduces to $y^{x^2-3} + \left(\frac{1}{y}\right)^{x^2-3} = 10$

Again let $y^{x^2-3} = t$. Then,

$$t + \frac{1}{t} = 10 \Rightarrow t^2 - 10t + 1 = 0$$

$$\Rightarrow \quad t = \frac{10 \pm \sqrt{100-4}}{2} = \frac{10 \pm \sqrt{96}}{2}$$

$$= \frac{10 \pm 4\sqrt{6}}{2} = 5 \pm 2\sqrt{6}$$

$\therefore\ (5+2\sqrt{6})^{x^2-3} = 5 \pm 2\sqrt{6} = (5+2\sqrt{6})^{\pm 1}$

$\Rightarrow x^2 - 3 = 1$ or $x^2 - 3 = -1$

$\Rightarrow x^2 = 4$ or $x^2 = 2$

$\Rightarrow x = \pm 2$ or $x = \pm\sqrt{2}$

27. Given, α, β, γ are the roots of the $2x^3 - 3x^2 + 6x + 1 = 0$.

Since $S_1 = -\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} \Rightarrow S_1 = \alpha + \beta + \gamma = -\left(\dfrac{-3}{2}\right) = \dfrac{3}{2}$

$S_2 = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3} \Rightarrow S_2 = \alpha\beta + \beta\gamma + \alpha\gamma = \dfrac{6}{2} = 3$

$S_3 = -\dfrac{\text{Coefficient of constant term}}{\text{Coefficient of } x^3} \Rightarrow S_3 = \alpha\beta\gamma = -\dfrac{1}{2}$

Now, $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= \left(\dfrac{3}{2}\right)^2 - 2 \times 3 = \dfrac{9}{4} - 6 = \mathbf{\dfrac{-15}{4}}.$

28. Given, α, β, γ are the roots of the equation $x^3 + 4x + 1 = 0$.

$\therefore S_1 = \alpha + \beta + \gamma = 0$ (Coefficient of $x^2 = 0$)

$S_2 = \alpha\beta + \beta\gamma + \alpha\gamma = 4$

$S_3 = \alpha\beta\gamma = -1$

$\therefore (\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1}$

$= \dfrac{1}{\alpha+\beta} + \dfrac{1}{\beta+\gamma} + \dfrac{1}{\gamma+\alpha}$

$= \dfrac{1}{-\gamma} + \dfrac{1}{-\alpha} + \dfrac{1}{-\beta}$

$\left[\begin{array}{l} \because \alpha + \beta + \gamma = 0 \\ \Rightarrow \alpha + \beta = -\gamma, \beta + \gamma = -\alpha, \gamma + \alpha = -\beta \end{array}\right]$

$= -\left[\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}\right] = -\left[\dfrac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}\right] = -\left[\dfrac{4}{-1}\right] = \mathbf{4.}$

29. Let $(\alpha - d)$, α, $(\alpha + d)$ be the three roots of the given cubic equation $x^3 - 12x^2 + 12x - 28 = 0$

$\therefore S_1 = (\alpha - d) + \alpha + (\alpha + d) = 12$

$\Rightarrow 3\alpha = 12 \Rightarrow \alpha = 4$

and $S_3 = (\alpha - d) . \alpha . (\alpha + d) = 28$

$\Rightarrow (4 - d) . 4 . (4 + d) = 28$

$\Rightarrow 16 - d^2 = 7 \Rightarrow d^2 = 9 \Rightarrow d = \pm\mathbf{3}.$

30. Let α, β be the roots of the equation $3ax^2 + 3bx + c = 0$. We have to find the equation whose roots are 3α and 3β, which can be got by putting $y = 3x$ in the given equation, *i.e.*, substituting x for $\dfrac{y}{3}$ in the given equations.

$\therefore$ The required equation is : $3a\left(\dfrac{y}{3}\right)^2 + 3b\left(\dfrac{y}{3}\right) + c = 0$

$\Rightarrow \dfrac{ay^2}{3} + by + c = 0 \quad \Rightarrow \quad ay^2 + 3by + 3c = 0$

$\Rightarrow \quad \boldsymbol{ax^2 + 3bx + 3c = 0.}$

31. As α, β are the roots of the equation $ax^2 + bx + c = 0$, so

$\alpha + \beta = -\dfrac{b}{a}, \alpha\beta = \dfrac{c}{a}$

The equation whose roots are $\dfrac{1-\alpha}{\alpha}$ and $\dfrac{1-\beta}{\beta}$ can be written as :

$x^2 - \left(\dfrac{1-\alpha}{\alpha} + \dfrac{1-\beta}{\beta}\right)x + \left(\dfrac{1-\alpha}{\alpha} . \dfrac{1-\beta}{\beta}\right) = 0$

Now, $\dfrac{1-\alpha}{\alpha} + \dfrac{1-\beta}{\beta} = \dfrac{\beta - \alpha\beta + \alpha - \alpha\beta}{\alpha\beta} = \dfrac{\alpha + \beta - 2\alpha\beta}{\alpha\beta} = \dfrac{\alpha+\beta}{\alpha\beta} - 2$

$= \dfrac{-b/a}{c/a} - 2 = -\dfrac{b}{c} - 2 = -\dfrac{b-2c}{c}$

and $\left(\dfrac{1-\alpha}{\alpha} . \dfrac{1-\beta}{\beta}\right) = \dfrac{1 - \alpha - \beta + \alpha\beta}{\alpha\beta} = \dfrac{1 - (\alpha+\beta) + \alpha\beta}{\alpha\beta}$

$= \dfrac{1}{\alpha\beta} - \dfrac{\alpha+\beta}{\alpha\beta} + 1 = \dfrac{a}{c} + \dfrac{b}{c} + 1 = \dfrac{a+b+c}{c}$

$\therefore$ The required equation is

$x^2 - \left[-\dfrac{(b+2c)}{c}\right]x + \left[\dfrac{a+b+c}{c}\right] = 0$

$\Rightarrow cx^2 + (b + 2c)x + (a + b + c) = 0$ ...(*i*)

Compairing eqn. (*i*) with the given equation $px^2 + qx + r = 0$, we get $r = \boldsymbol{a + b + c.}$

32. To find the equation whose roots are the negatives of the roots of the given equation, we replace x by $(-x)$ in the given equation.

$\therefore$ Required equation is

$(-x)^7 + 3(-x)^5 + (-x)^3 - (-x)^2 + 7(-x) + 2 = 0$

i.e, $-x^7 - 3x^5 - x^3 - x^2 - 7x + 2 = 0$

i.e., $x^7 + 3x^5 + x^3 + x^2 + 7x - 2 = 0.$

33. As, α, γ are the roots of the equation $Ax^2 - 4x + 1 = 0$, so

$\alpha + \gamma = \dfrac{4}{A}$ ...(*i*)

and $\alpha\gamma = \dfrac{1}{A}$ ...(*ii*)

Given, β, δ are the roots of the equation $Bx^2 - 6x + 1 = 0$,

so $\beta + \delta = \dfrac{6}{B}$ ...(*iii*)

and $\beta\delta = \dfrac{1}{B}$ ...(*iv*)

Given, α, β, γ, δ are in H.P., so

$\beta = \dfrac{2\alpha\gamma}{\alpha+\gamma} = \dfrac{2/A}{4/A} = \dfrac{1}{2}$

$\gamma = \dfrac{2\beta\delta}{\beta+\delta} = \dfrac{2/B}{6/B} = \dfrac{1}{3}.$

Also β is the root of the equation $Bx^2 - 6x + 1 = 0$, so

$B\beta^2 - 6\beta + 1 = 0 \Rightarrow B \times \dfrac{1}{4} - 6 \times \dfrac{1}{2} + 1 = 0$

$\Rightarrow \quad \dfrac{B}{4} - 2 = 0 \Rightarrow \dfrac{B}{4} = 2 \Rightarrow B = 8$

Given, γ is the root of the equation $Ax^2 - 4x + 1 = 0$, so

$A\gamma^2 - 4\gamma + 1 = 0$

$\Rightarrow A \times \dfrac{1}{9} - 4 \times \dfrac{1}{3} + 1 = 0 \quad \Rightarrow \quad \dfrac{A}{9} - \dfrac{1}{3} = 0 \Rightarrow A = \mathbf{3.}$

A, B = **3, 8** respectively.

34. The given equation is $(x - a)(x - b) = c$

$\Rightarrow x^2 - (a + b)x + (ab - c) = 0$

As α, β are the roots of this equation, so

$\alpha + \beta = a + b$ and $\alpha\beta = ab - c$

Let γ, δ be the roots of the equation $(x - \alpha)(x - \beta) + c = 0$

i.e., γ, δ are the roots of the equation $x^2 - (\alpha + \beta)x + (\alpha\beta + c) = 0$

$\therefore \quad \gamma + \delta = \alpha + \beta = a + b \quad ...(i)$

$\gamma\delta = \alpha\beta + c = ab - c + c = ab \quad ...(ii)$

$\therefore$ From (*i*) and (*ii*) we can infer that the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are $\boldsymbol{a}$ and $\boldsymbol{b}$.

35. Let the roots of the equation $x^3 - ax^2 + bx - c = 0$ be $(\alpha - 1), \alpha, (\alpha + 1)$

$\therefore S_2 = (\alpha - 1)\alpha + \alpha(\alpha + 1) + (\alpha + 1)(\alpha - 1) = b$

$\Rightarrow \alpha^2 - \alpha + \alpha^2 + \alpha + \alpha^2 - 1 = b$

$\Rightarrow 3\alpha^2 - 1 = b$

$\therefore$ Minimum value of $b = -1$, when $\alpha = \mathbf{0}$.

36. The given equations are written as :

$x^2 + 2ax + a^2 - 1 = 0 \quad ...(i)$

$x^2 + 2bx + b^2 - 1 = 0 \quad ...(ii)$

If α is the common root of both the equations, α satisfies both the equations, so,

$\alpha^2 + 2a\alpha + (a^2 - 1) = 0 \quad ...(iii)$

$\alpha^2 + 2b\alpha + (b^2 - 1) = 0 \quad ...(iv)$

Solving equations (*iii*) and (*iv*) simultaneously

$$\frac{\alpha^2}{2a(b^2-1) - 2b(a^2-1)} = \frac{\alpha}{(a^2-1)-(b^2-1)} = \frac{1}{2b - 2a}$$

$$\Rightarrow \frac{\alpha^2}{2ab^2 - 2ba^2 + 2(b-a)} = \frac{\alpha}{(a^2 - b^2)} = \frac{1}{2(b-a)}$$

$$\Rightarrow \frac{\alpha^2}{2ab\,(b-a) + 2(b-a)} = \frac{\alpha}{-(a+b)(b-a)} = \frac{1}{2(b-a)}$$

$$\Rightarrow \frac{\alpha^2}{2(b-a)(ab+1)} = \frac{\alpha}{-(a+b)(b-a)} = \frac{1}{2(b-a)}$$

[By the rule of cross-multiplication, the solution of two simultaneous equations :

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$ is

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}]$$

$\therefore \alpha^2 = ab + 1$ and $\alpha = -\frac{1}{2}(a + b)$

$\Rightarrow (ab + 1) = \frac{1}{4}(a + b)^2$

$\Rightarrow a^2 + b^2 + 2ab = 4ab + 4$

$\Rightarrow a^2 + b^2 - 2ab = 4 \Rightarrow (a - b)^2 = 4 \Rightarrow |a - b| = \mathbf{2}$.

37. Given equation is $x^3 - 3x + 11 = 0$

If α, β, γ are the roots of the given equation, then

$S_1 = \alpha + \beta + \gamma = 0$

$\Rightarrow \beta + \gamma = -\alpha, \gamma + \alpha = -\beta$ and $\alpha + \beta = -\gamma$

$\therefore$ The equation whose roots are $(\alpha + \beta), (\beta + \gamma), (\gamma + \alpha)$ is the equation whose roots are $-\gamma, -\alpha, -\beta$.

$\therefore$ We can obtain the required equations by replacing x by $(-x)$ in the given equation.

$\therefore$ Required equation is $(-x)^3 - 3(-x) + 11 = 0$

i.e, $-x^3 + 3x + 11 = 0$

i.e., $x^3 - 3x - 11 = \mathbf{0}$.

38. As α, β are the roots of the equation $ax^2 + bx + c = 0$, so

$\alpha + \beta = -\frac{b}{a}, \ \alpha\beta = \frac{c}{a}$

Also, $(\alpha + x), (\beta + x)$ are the roots of the equation

$px^2 + qx + r = 0$, then $\alpha + x + \beta + x = -\frac{q}{p}$

and $(\alpha + x)(\beta + x) = \frac{r}{p} \quad \Rightarrow \alpha + \beta + 2x = -\frac{q}{p}$

$\Rightarrow \frac{-b}{a} + 2x = -\frac{q}{p} \Rightarrow \boldsymbol{K = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)}$.

39. Let $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + ... \infty}}}$

$\Rightarrow x = \sqrt{6 + x}$

$\Rightarrow x^2 = 6 + x \Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow x = 3$ or -2

$\Rightarrow \boldsymbol{x = 3}$ as $x = -2$ does not satisfy the given equation.

40. The equation is $(x - a)(x - a - 1) + (x - a - 1)(x - a - 2) + (x - a)(x - a - 2) = 0$.

Let $(x - a) = y$, then the equation becomes

$y(y - 1) + (y - 1)(y - 2) + y(y - 2) = 0$

$\Rightarrow y^2 - y + y^2 - 3y + 2 + y^2 - 2y = 0$

$\Rightarrow 3y^2 - 6y + 2 = 0$

$\therefore$ Discriminant $= D = b^2 - 4ac = 36 - 4 \times 3 \times 2$

$= 36 - 24 = 12 > 0$

$\therefore$ Roots are real and distinct.

SELF ASSESSMENT SHEET

1. If the sum as well as the product of roots of a quadratic equation is 9, then the equation is:

(*a*) $x^2 + 9x - 18 = 0$ (*b*) $x^2 - 18x + 9 = 0$

(*c*) $x^2 + 9x + 9 = 0$ (*d*) $x^2 - 9x + 9 = 0$.

(*CDS 2010*)

2. If one root of the equation $\frac{x^2}{a} + \frac{x}{b} + \frac{1}{c} = 0$ is reciprocal of the other, then which of the following is correct ?

(*a*) $a = b$ (*b*) $b = c$ (*c*) $ac = 1$ (*d*) $a = c$

(*CDS 2012*)

3. If one root of the equation $ax^2 + x - 3 = 0$ is -1, then what is the other root ?

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{4}$ (*d*) 1

(*CDS 2010*)

4. If the equation $(a^2 + b^2) x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ has equal roots, then which one of the following is correct ?

(*a*) $ab = cd$ (*b*) $ad = bc$
(*c*) $a^2 + c^2 = b^2 + d^2$ (*d*) $ac = bd$ **(*CDS 2010*)**

5. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then what is the value of $\alpha^3 + \beta^3$?

(*a*) $\frac{b^3 + 3abc}{a^3}$ (*b*) $\frac{a^3 - b^3}{3abc}$ (*c*) $\frac{3abc - b^3}{a^3}$ (*d*) $\frac{b^3 - 3abc}{a^3}$

(*CDS 2008*)

6. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of their squares, then which one of the following is correct ?

(*a*) $a^2 + b^2 = c^2$ (*b*) $a^2 + b^2 = a + b$
(*c*) $2ac = ab + b^2$ (*d*) $2c + b = 0$

7. One root of $x^2 + kx - 8 = 0$ is the square of the other, then the value of k is :

(*a*) 2 (*b*) 8 (*c*) –8 (*d*) –2

(*CAT 1995*)

8. Let p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$. What is the minimum possible value of $p^2 + q^2$?

(*a*) 0 (*b*) 3 (*c*) 4 (*d*) 5

(*CAT 2003*)

9. If α, β, γ are the roots of $x^3 - 2x^2 + 3x - 4 = 0$, then the value of $\alpha^2 \beta^2 + \beta^2 \gamma^2 + \gamma^2 \alpha^2$ is :

(*a*) –7 (*b*) –5 (*c*) –3 (*d*) 0

(*EAMCET 2007*)

10. If α, β are the roots of the equations $x^2 + 4x + 3 = 0$, then the equation whose roots are $2\alpha + \beta$ and $\alpha + 2\beta$ is

(*a*) $x^2 - 12x - 33 = 0$ (*b*) $x^2 - 12x + 35 = 0$
(*c*) $x^2 + 12x - 33 = 0$ (*d*) $x^2 + 12x + 35 = 0$.

(*J&K CET 2009*)

11. The equation whose roots are twice the roots of the equation $x^2 - 3x + 3 = 0$ is

(*a*) $x^2 - 3x + 6 = 0$ (*b*) $x^2 - 4x + 8 = 0$
(*c*) $x^2 - 6x + 12 = 0$ (*d*) $x^2 - 8x + 6 = 0$

12. If $x^2 + mx + n = 0$ and $x^2 + px + q = 0$ have a common root, then the common root is

(*a*) $\frac{q - n}{m - p}$ (*b*) $\frac{q - n}{m + p}$
(*c*) $\frac{q + n}{m + p}$ (*d*) None of these

ANSWERS

1. (*d*)	**2.** (*d*)	**3.** (*c*)	**4.** (*b*)	**5.** (*c*)	**6.** (*c*)	**7.** (*d*)	**8.** (*d*)	**9.** (*a*)	**10.** (*d*)
11. (*c*)	**12.** (*a*)								

HINTS AND SOLUTIONS

1. Equation : x^2 – (Sum of roots)x + Product of roots = 0

$\Rightarrow \mathbf{x^2 - 9x + 9 = 0.}$

2. The equation can be written as $\frac{1}{a}x^2 + \frac{1}{b}x + \frac{1}{c} = 0.$

i.e., $bcx^2 + acx + ab = 0$.

Let α, $1/\alpha$ be the roots of the given equation, then

product of roots $= \alpha \times \frac{1}{\alpha} = \frac{ab}{bc} \Rightarrow \frac{ab}{bc} = 1 \Rightarrow \boldsymbol{a = c.}$

3. Let the other root of the equation $ax^2 + x - 3 = 0$ be α.

As (–1) is a root of the given equation, it satisfies the given equation, *i.e.*, $a(-1)^2 + (-1) - 3 = 0 \Rightarrow a - 1 - 3 = 0 \Rightarrow a = 4.$

$\therefore$ The equation becomes $4x^2 + x - 3 = 0$.

Now, product of roots $= \alpha \times (-1) = -\frac{3}{4}$

$\therefore \alpha = \mathbf{\frac{3}{4}.}$

4. Equal roots $\Rightarrow$ Discriminant = 0

$\Rightarrow 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$

$\Rightarrow [\cancel{a^2c^2} + \cancel{b^2d^2} + 2acbd] - [\cancel{a^2c^2} + b^2c^2 + a^2d^2 + \cancel{b^2d^2}] = 0$

$\Rightarrow -[b^2c^2 + a^2d^2 - 2acbd] = 0$

$\Rightarrow (bc - ad)^2 = 0 \Rightarrow \boldsymbol{bc = ad.}$

5. $\alpha + \beta = -b/a$, $\alpha\beta = c/a$.

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$= (\alpha + \beta)\{(\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta\}$

$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$

$= -\frac{b}{a}\left[\frac{b^2}{a^2} - \frac{3c}{a}\right]$

$= \frac{-b^3}{a^3} + \frac{3bc}{a^2} = \boldsymbol{\frac{3abc - b^3}{a^3}}$

6. Let α, β be the roots of the equation $ax^2 + bx + c = 0$. Then,

$\alpha + \beta = -b/a$, $\alpha\beta = c/a$

Given, $\alpha + \beta = \alpha^2 + \beta^2$

i.e., $\alpha + \beta = (\alpha^2 + \beta)^2 - 2\alpha\beta$

$\Rightarrow \frac{-b}{a} = \frac{b^2}{a^2} - \frac{2c}{a}$

$\Rightarrow -\frac{ab}{a^2} = \frac{b^2}{a^2} - \frac{2ac}{a^2} \Rightarrow ab + b^2 = \mathbf{2ac.}$

7. Let α and α^2 be the roots of the equation $x^2 + kx - 8 = 0$.

Then, product of roots $= \alpha \cdot \alpha^2 = -8$

$\alpha^3 = -8 \Rightarrow \alpha = -2$

$\therefore$ The root (–2) satisfies the given equation, *i.e.*,

$(-2)^2 + k \cdot (-2) - 8 = 0$

$4 - 2k - 8 = 0 \Rightarrow -2k = 4 \Rightarrow k = \mathbf{-2.}$

8. If p and q are the roots of the equation
$x^2 - (\alpha - 2)x - (\alpha + 1) = 0$.
$\therefore$ Sum of roots $= p + q = (\alpha - 2)$
Product of roots $= pq = -\alpha - 1$
$\therefore p^2 + q^2 = (p + q)^2 - 2pq$
$= (\alpha - 2)^2 + 2(\alpha + 1)$
$= \alpha^2 + 4 - 4\alpha + 2\alpha + 2 = (\alpha + 1)^2 + 5$
$p^2 + q^2$ will be minimum when $\alpha = 0$.
$\therefore$ Minimum value of $p^2 + q^2 = \mathbf{5}$.

9. Given, α, β, γ are the roots of $x^3 - 2x^2 + 3x - 4 = 0$. Then,
$S_1 = \alpha + \beta + \gamma = 2$
$S_2 = \alpha\beta + \beta\gamma + \gamma\alpha = 3$
$S_3 = \alpha\beta\gamma = 4$
Now, $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$
$= \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta^2\gamma + 2\beta\gamma^2\alpha + 2\alpha^2\beta\gamma$
$= \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\beta + \gamma + \alpha)$
$\Rightarrow \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$
$= 3^2 - 2 \times 4 \times 2 = 9 - 16 = \mathbf{-7}$.

10. Given, α, β are the roots of the quadratic equations $x^2 + 4x + 3 = 0$,
$\Rightarrow$ Sum of roots $= \alpha + \beta = -4$ ...(i)
Product of roots $= \alpha\beta = 3$ ...(ii)
Given, $2\alpha + \beta$ and $\alpha + 2\beta$ are the roots of the required equation, so
Required equation is
$= x^2 - \underbrace{(2\alpha + \beta + \alpha + 2\beta)}_{\text{Sum of roots}}x + \underbrace{(2\alpha + \beta)(\alpha + 2\beta)}_{\text{Product of roots}} = 0$
Sum $= 2\alpha + \beta + \alpha + 2\beta = 3\alpha + 3\beta = 3(\alpha + \beta)$
$= 3 \times -4 = -12$
Product $= (2\alpha + \beta)(\alpha + 2\beta) = 2\alpha^2 + \beta\alpha + 4\alpha\beta + 2\beta^2$
$= 2(\alpha^2 + \beta^2) + 5\alpha\beta$
$= 2(\alpha^2 + \beta^2 + 2\alpha\beta) + \alpha\beta$
$= 2(\alpha + \beta)^2 + \alpha\beta = 2 \times 16 + 3 = 35$
$\therefore$ Reqd. equation is $\boldsymbol{x^2 - 12x + 35 = 0}$.

11. Let α, β be the roots of the given equation $x^2 - 3x + 3 = 0$.
Given, $\alpha + \beta = +3$, $\alpha\beta = 3$
By the given condition, the roots of the required equation are 2α and 2β.
$\therefore$ Sum of roots $= 2\alpha + 2\beta = 2(\alpha + \beta) = 2 \times 3 = 6$
Product of roots $= 2\alpha \,.\, 2\beta = 4\alpha\beta = 4 \times 3 = 12$.
$\therefore$ Required equation is $\boldsymbol{x^2 - 6x + 12 = 0}$

12. Let α be the common root of the equations $x^2 + mx + n = 0$ and $x^2 + px + q = 0$. Then,
$\alpha^2 + m\alpha + n = 0$ ...(i)
$\alpha^2 + p\alpha + q = 0$ ...(ii)
Solving (i) and (ii) simultaneously, we get

$$\frac{\alpha^2}{mq - np} = \frac{\alpha}{n - q} = \frac{1}{p - m} \Rightarrow \alpha = \frac{n - q}{p - m} = \boldsymbol{\frac{q - n}{m - p}}.$$

WHY MODULUS JUNIOR COLLEGE ?

Best IITJEE Faculty : Team led by an Alumni of IIT Delhi,**MODULUS** has expert faculty to provide IITJEE coaching, mentoring and counselling to the students to reach their goal.

Exceptional Teaching Methodology : Concept Oriented Teaching rather than Rote Learning. Sufficient self-study time is given to grasp the concepts and to apply them to solve the difficult problems. Classes will be interactive to keep students curious and interested in subject.

"I listen - I forget,
I read -I remember,
I do - I understand"

following this saying, measures like asking students to teach some subtopic will be taken.

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MODULUS Junior College

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7th

For 7th Class Students

ENRICH TWO YEAR

8th

For 8th Class Students

ENRICH ONE YEAR

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For 9th Class Students

REFINE TWO YEAR

10th

For 10th Class Students

REFINE ONE YEAR

The goal of this program is to lay strong foundation for the most prestigious competitive exams like NTSE, IIT-JEE, KVPY, NSEJS and other Olympiads.
This also helps students to perform exceptionally in board exams.

11th

For 11th Class Students

TRIUMPH TWO YEAR

12th

For 12th Class Students

TRIUMPH ONE YEAR

The above programs will help the student to perform well in IIT-JEE Mains and Advanced exams. This also equip the student excel in other engineering and board exams.

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